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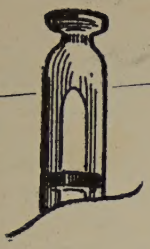

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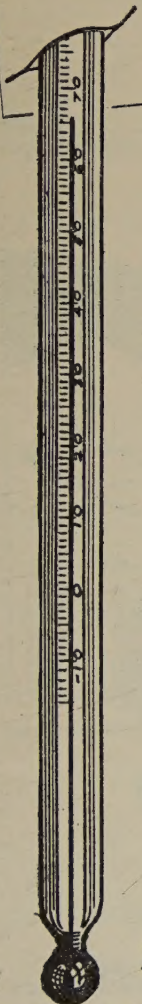

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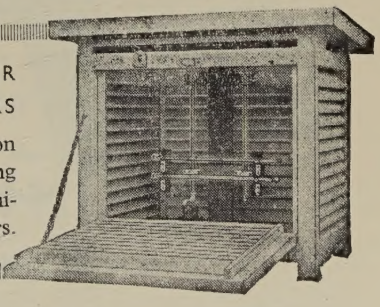

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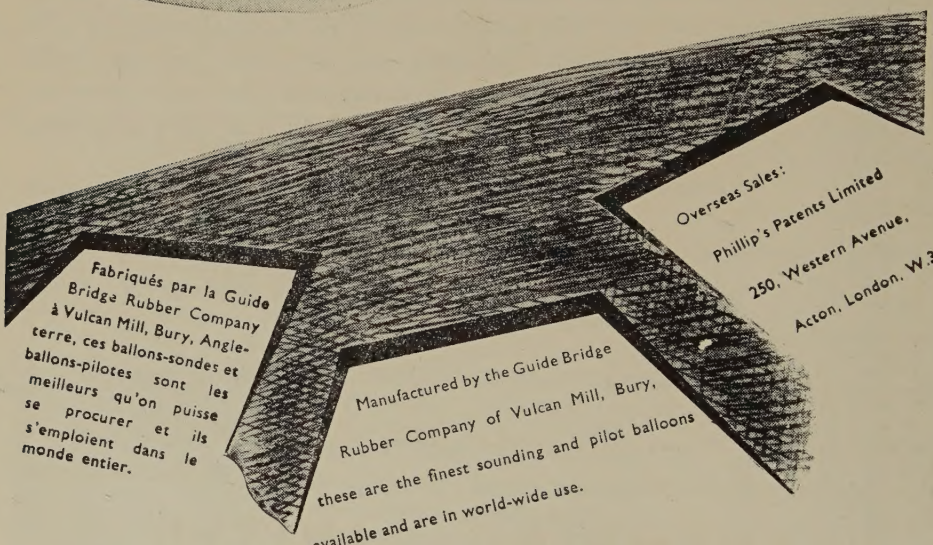

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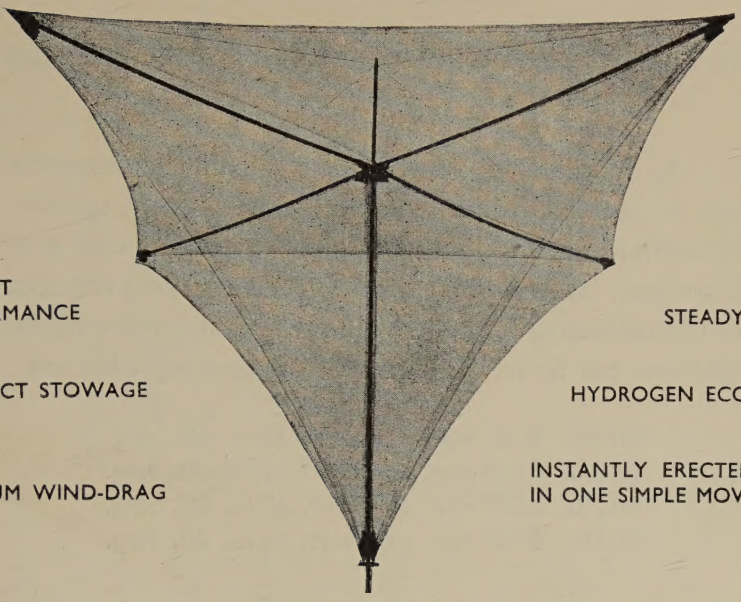

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THE AIRFLOW OVER MOUNTAINS

A review of the state of current knowledge

By G. A. CORBY
Meteorological Office, Dunstable

(Manuscript received 21 July 1954, in revised form 19 August 1954)

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SUMMARY

A brief survey is first given of the observational evidence regarding special airflow effects in the neighbourhood of mountains, as provided by the visual evidence of clouds, the experiences of glider pilots and effects noted by the pilots of powered aircraft. This serves to indicate the phenomena requiring explanation. The review of theoretical studies is confined mainly to the work of Queney and Scorer. Both these authors apply the perturbation method to laminar, isentropic and inviscid flow over an ideal ridge. Queney deals with a uniform airstream (constant velocity and constant stability) and derives streamlines showing wave motions, both vertical and horizontal. Queney's waves are dependent partly on the stability and partly on geostrophic forces, and the order of the wavelength is several hundred km. Scorer, in his treatment, deals with smaller-scale phenomena, so enabling geostrophic forces to be neglected, and provides for variations of stability and wind shear in the vertical. He finds that realistic wave effects, of the type observed in nature, are predicted by the theory provided the variation of stability and wind shear with height is in accord with

certain criteria. The criteria have some practical value for forecasting purposes. Past experimental work with models in wind tunnels is briefly considered in the light of what can be deduced from theoretical treatments regarding the conditions for dynamical similarity. More recent work by Long, using a tank containing several layers of immiscible liquid to simulate a density gradient is referred to briefly; this technique appears to have promise. A comprehensive field study by Förchtgott, who used teams of gliders, is summarized. From his observations, Förchtgott has built up a theory of the flow patterns which occur in the lee of mountains. The main value of Förchtgott's work lies in the practical support it provides for some of the results deduced by perturbation theory and in throwing light on the nature of non-laminar flow patterns which cannot be dealt with by existing theoretical treatments. In a concluding section, aviation applications of existing knowledge on this subject are referred to briefly.

1. Introduction

It has long been known that the airflow is commonly more disturbed over mountainous terrain than over level country. In the past, information on the subject in meteorological textbooks has usually been scanty and generally confined to the statement that disturbances to the airstream above a given mountain can be expected to extend to about three times its height. This gives no clue as to the nature of the disturbances or the meteorological factors which have a bearing on them. From its very incompleteness, therefore, the information is quite unsatisfactory expecially now, in view of the increasing demand for more detailed advice on the subject and the better understanding of such problems which has developed recently.

It might be supposed that the effect of a mountain range on an airstream would be merely to bring about a change in the extent, scale and degree of turbulence experienced by an aircraft flying above. Turbulence, however, is essentially a disorganized type of flow whilst often the airstream over a mountain is deformed in a fully organized manner which is highly significant but not necessarily turbulent. In this review we shall be much concerned with such organized deformations and the circumstances in which they occur.

During the last few years, there has grown a considerable body of experimental data on the subject from various sources and notable theoretical researches into the problem have been carried out, so much so that over at least an important part of the subject a logical system of ideas has developed and can now be presented. As glider pilots rely on vertical currents to support their sailplanes, it is natural that they should have learnt a great deal of the special airflow effects which occur in the neighbourhood of mountains; accordingly, much of the practical evidence has come from glider pilots. Förchtgott has exploited the potentialities of gliders in a valuable and illuminating programme of field research. More recently the experiences of the pilots of powered aircraft have been receiving attention and have been reported by Austin, Turner and others. As regards other specific experimental work, mention must be made of the investigation by Field and Warden into the air currents in the Bay of Gibraltar, the work of Abe in Japan who studied mountain cloud formations and supplemented this by wind-tunnel experiments, and Küttner's studies of the airflow over the Alps. Currently, the powerful mountain-wave phenomena at Bishop, California, are the subject of an elaborate investigation as part of the U.S. Research Project on this subject.

The challenge to the mathematician offered by the problem of mountain airflow has been taken up, and major contributions in the form of theoretical treatments have been made by Lyra, Queney, Scorer and others. The hydrodynamical theory solves the problem of the flow pattern successfully if the flow is laminar and steady, but when this is not the case we must draw what conclusions we can from the observational data. The theory indicates the great limitations of wind tunnel experiments and makes it clear that the problem of ensuring dynamical similarity in the general case poses almost insuperable laboratory difficulties; any past results obtained in this way, therefore, must be interpreted with great caution.

It is the purpose of this review to summarize existing knowledge on the subject of mountain airflow. The literature is very extensive and we shall confine ourselves to those aspects which appear to have a bearing, real or potential, on the aviation problems arising from the subject.

2. Summary and examples of observational evidence

The observational evidence of special airflow effects in the neighbourhood of mountains, notably the phenomenon of stationary lee waves, falls roughly into three categories, viz., the visual evidence provided by clouds, the experiences of glider pilots, and effects observed from powered aircraft. It will be convenient, before discussing theoretical and other studies, to summarize this evidence and give a few examples, in order to make clear the nature of the phenomena requiring explanation.

(*a*) *The evidence of mountain clouds*

Lenticular clouds are frequently seen over hilly or mountainous terrain in all parts of the world. As observed from the ground they have smooth lens-shaped outlines and remain stationary relative to the ground or nearly so. The wind blows through them and this can be verified when the movement of an identifiable fragment of the cloud can be followed visually. This has been confirmed by time-lapse photography of mountain clouds, for example by Abe (1932). The inference that such clouds form where the airstream executes a vertical undulation initiated by the terrain below is almost inescapable. On many occasions, especially to the lee of long ridges, a succession of lenticular clouds, parallel to each other and to the ridge, may be seen; the suggestion that they constitute visible signs of systems of atmospheric lee waves is very strong. Over irregular hilly terrain, the lenticulars too are irregularly distributed; such skies are frequent in Scotland and many examples are to be found in the Clarke collection of cloud photographs.

A similar phenomenon can sometimes be observed when an airstream containing a layer of stratocumulus crosses a mountainous area. Organised stationary clearances of the cloud in the form of holes or parallel zones to the lee of hills or ridges can be seen. The inference is that the clearances are located where the airstream executes downward undulations associated with the topography.

A number of field studies have been carried out aimed at elucidating the mechanism causing particular cloud formations which are characteristic of given hills or mountains. Amongst these are the exhaustive study by Manley (1945) of the Crossfell Helm wind, and its associated clouds, and the study by Küttner (1938) of the so-called Moazagotl clouds which form over the Reisengebirge in the Alps. Although on a different scale, both these phenomena have much in common and in particular at both locations rotor clouds are commonly observed at levels similar to those of the hilltops, whilst lenticular clouds may also be seen high above the rotors.

Recently Ludlam (1952) has discussed the formation of orographic cirrus over the hills of the United Kingdom and has put forward evidence which strongly supports his suggestion that a great deal of cirrus forms in atmospheric waves initiated by topographical features. He is satisfied from his observations that hills in the United Kingdom can produce waves having sufficient amplitude at 20,000 ft or above to produce cirrus cloud. This comes as no surprise to those who have accepted that ' mother of pearl ' clouds seen high above the Norwegian mountains at 70,000-80,000 ft and described by Störmer (1948) also owe their origin to topography. ' Mother of pearl ' clouds have been observed in other parts of the world besides Norway, even it seems over Scotland on rare occasions.

(b) *The experiences of glider pilots*

In the early days of gliding, the sources of vertical motion made use of were first the upslope motion to be found on the windward side of hills and ridges and later thermal up-currents. During the 1930's, the possibilities of soaring in atmospheric waves began to be exploited, especially on the Continent. Numerous ascents were made on the north side of the Alps in föhn-wind conditions and by 1939 several glider ascents to beyond 30,000 ft had been made in the air rising on the upwind side of wave clouds. The significant aspect was that many of these ascents were made well to the lee of the highest ground and could not, therefore, have been simple cases of flights in the airstream ascending the windward slopes. The first important ascent by a glider in a standing wave in the United Kingdom was made in 1939 by McLean who reached 11,000 ft on the Helm phenomenon. On this first occasion, McLean had difficulty in getting down and was only able to do so by locating the downdraught of the wave. The occurrence was described graphically by Horsley (1945).

Since the war, wave soaring has become commonplace and the various gliding journals literally abound with articles by glider pilots describing experiences in standing waves. Some of these accounts are merely descriptive but many others are based on careful and detailed observation. The sinking speed of a glider for a given forward speed is a quantity which is known fairly exactly, expecially if the machine has been calibrated, and therefore gliders are able to explore waves and determine the vertical component of the wind. It is by such means that a fairly complete picture of the characteristics of standing lee waves has emerged.

From large numbers of reports of this sort has grown the conception of standing waves as vertical undulations in the airstream to the lee of hills and ridges. It is found that, over the lee ground, waves, if any, increase in amplitude upwards to some level above which the waves decrease and eventually die out. They die away downstream but a succession of several observable waves has often been noted in the United Kingdom. The order of magnitude of the wavelength is usually in the range 1 to 10 mi, but wavelengths up to 20 mi or so are sometimes observed.

We can scarcely leave this reference to gliders without mentioning the spectacular 'Bishop' wave which is characteristic of the airstream over the Sierra Nevada range in California during winter time. Many ascents to well above 40,000 ft have been made by gliders in this wave system and there seems no reason why a pressurized glider, which is planned, should not reach far greater heights. If the wave develops at all, vertical components of 2,000 ft/min are common, whilst 4,000 ft/min has been recorded and it is believed that components up to 5,000 ft/min may occur on occasions. The field work for the U.S. Mountain Wave Project was carried out at this location, but reports published so far have been confined to descriptions of the organization of the project and of the techniques used for tracking gliders, etc. A reasoned discussion of the results obtained has yet to appear.

Another range of mountains which provide wave effects on a powerful scale is to be found in New Zealand's South Island. In the vicinity of Christchurch some remarkable ascents by gliders have been made in lee waves initiated by the Southern Alps, some 60-70 mi to the north-west. One such ascent to 19,000 ft has been described by Evans (1954) and on the occasion in question the lift at the highest part of the flight exceeded the maximum which the glider's variometer could indicate, even with the machine in a steep diving attitude. In fact, descent was only possible by flying out of the region of rising air. This location has much to recommend it as a site for field investigation, because the mountain range is quite isolated with ocean to the north-west and flat ground and further ocean to the south-east.

(c) *Effects observed from powered aircraft*

During the last three or four years there has been a steadily increasing number of reports of mountain airflow effects from the pilots of powered aircraft. This may be because aircraft are now flying over mountain ranges which lie across air routes, with increasing regularity and frequency. The reports confirm that areas of lift and sink are commonly to be found over and to the lee of mountains. To date most of these reports have been made by pilots who have for some reason a special interest in the subject as, for example, because they are also glider pilots, or because a specific experience was sufficiently notable to arouse a special interest. Mild standing lee-waves may easily pass unnoticed in a powered aircraft because they are not necessarily accompanied by any turbulence effects and in fact many reports mention the remarkably smooth flying conditions in waves. In addition, a pilot ignorant of wave effects might well ascribe gentle height fluctuations, having a period of a minute or two, to oscillations of the aircraft due to instability of trim or to the movement of passengers about the cabin.

Austin (1952) and Turner (1951, 1952) have reported cases of standing waves encountered by pilots. Furthermore, the aviation journals contain several contributions by pilots describing their experiences. One pilot, Capt. Neilan, late of Skyways, whose work amongst other things involved the flight testing of aircraft over Scotland, has reported details of several experiences in standing waves. On many occasions Capt. Neilan has encountered alternate areas of up and down currents exceeding 1,000 ft/min; such currents force themselves upon his attention because they are a hindrance to flight testing.

Capt. D. Mason of B.E.A.C. (1954) gave a detailed description of an incident over the mountains of northern Spain on 18 December 1952, when he was flying a Viking aircraft from Madrid to London. He approached the mountains north of Madrid at 11,000 ft and the aircraft subsequently fell to 8,800 ft and then rose to 14,000 ft. This was repeated three times in what were undoubtedly powerful standing waves. The height changes took place in spite of his using maximum power in the areas of sink and almost closing the throttles in the areas of rising air. It can be inferred from this that the vertical components exceeded 2,500 ft/min.

Some fifty or so reports of mountain-airflow effects have now been collected at Northolt from B.E.A.C. pilots and a record of these has been assembled by Turner in an unpublished report*. Three cases from this record are now quoted by way of example :

(i) 18 Dec. 1952, 2000 GMT. Mean flying height 8,500 ft. Location : midway between Nice and Montélimar. After slight vertical motion two powerful waves were encountered giving lift of 1,000 ft/min with aircraft in descending attitude and sink of 1,000 ft/min with full climbing power on. Wave crests about 8 min apart. Wind 340°, 30 kt. Sky cloudless.

(ii) 21 March, 1952, 1210-1310 GMT. Mean flying height 10,500 ft. Location : Maritime Alps into Rhone Valley. Successive up and down draughts of 1,500 ft/min resulting in 2,000 ft displacement of aircraft. There were 3/8 Cu below aircraft, otherwise no cloud. Wind NW, 40 kt.

(iii) 11 March, 1953, 1030 GMT. Mean flying height 5,500 ft. Location : Barrow-in-Furness to Solway. Smooth waves encountered giving up and down currents of 700 ft/min. Pilot made no attempt to counteract vertical motion, and therefore there were only small fluctuations of airspeed. About six separate waves were distinguished but beyond Solway the waves became negligible. Cloud : 6/8 Sc below aircraft and 7/8 Ac above. Wind 350°, 35-40 kt.

One feature worth noting is that in many cases the pilot records a marked drop in airspeed on entering an area of sink and an increase in airspeed in the areas of lift. This,

* These data are available for inspection in the library of the Meteorological Office, London.

of course, would be expected if the pilot uses his controls in an attempt to maintain constant height, but it is a little unexpected to find similar (although smaller) variations of airspeed when no such correcting action is taken. This aspect has been discussed from the aerodynamical standpoint by Yates (1951).

Turner examined the distribution of lapse rate and wind in the vertical in the cases reported at Northolt, with a view to ascertaining whether the airstreams possessed any special characteristics common to all cases. His conclusions were used to carry out a forecasting test which was described in an unpublished report. Briefly, he found that when pilots noted special airflow effects over hilly terrain, there was always a markedly stable layer (commonly at least isothermal) at middle levels, usually with a layer of steep lapse rate below. The latter lapse rate was most often between the wet and dry adiabatics, i.e., it was stable for unsaturated air, and in this connection the remarks in Section 3 (g) (ii) regarding the simultaneous existence of convection clouds and waves are relevant. The type of effect depended upon the wind shear. When standing waves were found, the vertical wind shear was in the direction of the general wind, i.e., the wind increased with height. In the cases of marked turbulence the vertical wind shear was opposed to the general wind, i.e., there was reversed wind shear. In some intermediate cases both turbulence and standing waves, sometimes at different levels, occurred. It cannot be held, of course, that this kind of empirical approach yields conclusive evidence of necessary conditions for given effects, but it is interesting to note that although based on few observations Turner's conclusions are broadly consistent with some of the results obtained by Scorer and Förchtgott and described later in this review. With regard to the lower layer of steep lapse rate, found by Turner, one might expect this to be a normal attribute of a fast-moving airstream over rough hilly terrain. However, he found the layer to be at least 5,000 ft thick and it may, therefore, have real significance. Turner's views developed from his own personal enquiries of pilots in the course of aviation forecasting, and his results showed that useful information may be obtained in this way.

3. Theoretical studies*

(*a*) *General*

The problem of the flow of an incompressible, homogeneous and inviscid current over an obstacle at the bottom was first solved by Kelvin in 1886. Lamb extended this work in 1916 to cover the case of a stratified, but still incompressible, fluid. In their solutions a system of stationary gravity waves appears downstream of the obstacle, provided the speed of the flow is less than a critical value depending on the depth of the fluid. These waves are analogous to those which constitute the wash of a ship.

The corresponding atmospheric problem is a great deal more complex. Firstly, apart from shallow adiabatic layers, e.g. near the ground on hot days, the atmosphere has continuous stable stratification. Secondly, the atmosphere, being compressible, has no definite upper boundary; as we shall see later the formulation of a satisfactory upper boundary condition in the atmospheric problem is difficult and has given rise to some controversy. Finally, for phenomena on the scale of, say, the airflow over the Alps, the earth's rotation must be taken into account, whilst on an even larger scale the variation of Coriolis parameter with latitude must be introduced.

So far, all theoretical treatments have made use of the perturbation method since there seems no prospect of achieving analytical solutions by any other approach. In this method, the motion is considered as a disturbance superimposed on a given basic flow

* For convenience in this section a common notation is used throughout which is not necessarily that of the particular author being quoted. After Queney we shall use $\rightleftharpoons$ to denote, ' is equal to the real part of.'

which is assumed to be laminar, frictionless and isentropic in the first instance. The hydrodynamical equations of motion are set up and are used with the equation of continuity and the equation expressing the isentropic nature of the flow. It is assumed that the disturbance quantities are small compared with the values in the basic motion and this justifies neglect of the squares and products of the disturbance quantities and enables the equations to be linearized. Elimination of all but one of the unknowns yields a partial second-order differential equation which is used to determine the nature of the perturbation.

(*b*) *The work of Queney (1947)*

We shall first consider the work of Queney who commenced studying the subject in 1936. After several contributions he published in 1947 a completely general theory of adiabatic perturbations in a stratified rotating atmosphere. The problem of the flow of an airstream over a mountain is a particular application of his generalized theory. Queney found that the degree of stability and the vertical component of the earth's rotation made the important contributions to the terms in the wave equation and he was able to justify neglect of several other terms. The detail of Queney's mathematics is elaborate and lengthy. For brevity in this review and for convenience in comparing subsequently Queney's results with Scorer's we may regard Queney's treatment as equivalent to the derivation of the wave equation represented by :

$$\left(1 - \frac{f^2}{k^2 U^2}\right) \frac{\partial^2 \psi}{\partial z^2} - \left(\frac{g}{c^2} + \beta\right) \frac{\partial \psi}{\partial z} + \left(\frac{g\beta}{U^2} - k^2\right) \psi = 0 \quad . \quad . \quad (1)$$

where U = horizontal velocity of the undisturbed current (assumed constant and uniform in space)

ψ = stream function of the disturbance

$f = 2 \times$ vertical component of the earth's angular velocity

g = acceleration due to gravity

β = coefficient of stability defined as $\frac{1}{\theta} \frac{\partial \theta}{\partial z}$ where θ is potential temperature

k = wave number of the disturbance in the x direction (the direction of U), i.e., a harmonic variation of the disturbance is assumed in the x-direction of wavelength $2\pi/k$.

c = velocity of sound

z = height measured vertically upwards.

This equation expresses the variation of the disturbance with height assuming that the variation in the horizontal is harmonic. The nature of the source of the perturbation is as yet unspecified. Queney introduces a further simplification by taking the stability as constant with height and is then able to treat the coefficients of Eq. 1 as sensibly constant (for a given value of k) provided the magnitude of the perturbation is not of the order of thousands of km.

$\sqrt{\frac{g\beta}{U}}$ is denoted by l_s and $\frac{f}{U}$ by l_f and we put $\mu = + k \sqrt{\frac{l_s^2 - k^2}{l_f^2 - k^2}}$.

The solution of Eq. (1) may then be written

$$\psi = A\, e^{\lambda_1 z} + B\, e^{\lambda_2 z}$$

where $\lambda_1, \lambda_2 = \frac{1}{2}\left\{\frac{k^2(g/c^2+\beta)}{k^2-l_f^2} \pm \sqrt{\left(\frac{k^2(g/c^2+\beta)}{k^2-l_f^2}\right)^2 + 4\mu^2}\right\} \simeq \frac{1}{2}(g/c^2+\beta) \pm \mu$

if k is not in the vicinity of l_f or l_s

Also if ρ = density, $(g/c^2+\beta) = -\frac{1}{\rho}\frac{\partial \rho}{\partial z}$

$= 2S$, say (Queney calls S the heterogeneity).

Thus the solution may be expressed as

$$\psi = e^{Sz}(A\,e^{\mu z} + B\,e^{-\mu z}) \quad . \quad . \quad . \quad . \quad (2)$$

Queney shows that with typical atmospheric values, e^{Sz} is closely equal to $\sqrt{\rho_0/\rho}$.

The vertical displacement of the streamlines from their undisturbed level will then be given by

$$\zeta = \frac{\psi}{U} = \frac{1}{U}\sqrt{\rho_0/\rho}\,(A\,e^{\mu z} + B\,e^{-\mu z}) \quad . \quad . \quad . \quad (3)$$

We have the two cases :

(i) If μ is real, i.e., if $k > l_s$ or $k < l_f$ we must have $A = 0$, otherwise the disturbance would increase exponentially upwards indefinitely, clearly an unacceptable solution because such a result is impractical.

(ii) If μ is imaginary, i.e. if $l_s > k > l_f$ the solution becomes

$$\zeta = \frac{1}{U}\sqrt{\rho_0/\rho}\,(A\,e^{i\nu z} + B\,e^{-i\nu z}) \text{ where } \mu = i\nu$$

This represents sinusoidal variation in the vertical. Relative to the airstream there is a downward progressing wave component $e^{i\nu z}$ and an upward component $e^{-i\nu z}$. Both are indeterminate for $z = \infty$ and so there is some difficulty about the upper boundary condition. Queney overcomes the difficulty by temporarily introducing small viscosity, an artifice used by Rayleigh and others in problems of fluid motion. He does this by applying a factor $e^{-\epsilon z}$ to the term in $e^{i\nu z}$ and $e^{+\epsilon z}$ to the term in $e^{-i\nu z}$, where ϵ is small and positive. He is then, of course, able to reject the upward progressing wave and takes as his solution for imaginary μ :

$$\zeta = \frac{A}{U}\sqrt{\frac{\rho_0}{\rho}}\,e^{i\nu z}$$

The second boundary condition at the ground causes no difficulty. If the ground profile is the sinusoidal corrugation represented by the real part of e^{ikx} then we must have $\zeta = e^{ikx}$ for $z = 0$. Thus the appropriate solutions for such a ground profile are :

For $k > l_s$ or $k < l_f$,

$$\zeta = \sqrt{\frac{\rho_0}{\rho}}\,e^{ikx-\mu z}$$

$$= \sqrt{\frac{\rho_0}{\rho}}\,e^{-\mu z}\cos kx \quad . \quad . \quad . \quad . \quad . \quad (4)$$

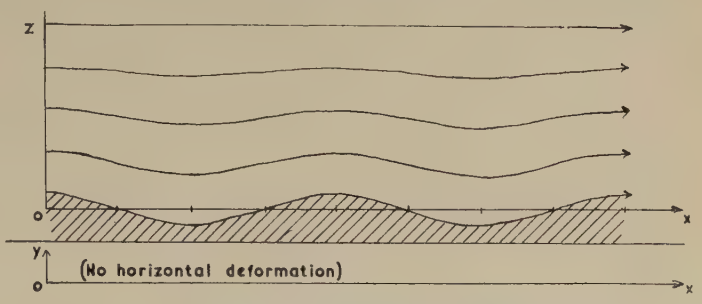
For $l_s > k > l_f$,

$$\zeta = \sqrt{\frac{\rho_0}{\rho}}\,e^{i(kx+\nu z)}$$

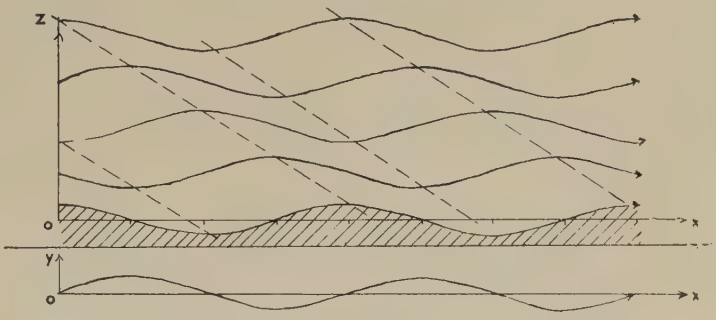
$$= \sqrt{\frac{\rho_0}{\rho}}\cos(kx+\nu z) \quad . \quad . \quad (5)$$

Similarly, expressions could be deduced for the horizontal disturbance and the variation of pressure or density. The concept of a sinusoidal ground profile extending to infinity does not in itself provide a solution of direct practical value but it is derived in anticipation of subsequent generalization by the Fourier integral method. Before doing so it is worth examining the elementary solutions given by Eqs. (4) and (5) and in Fig. 1 are illustrated the streamlines corresponding to steady flow over an infinite sinusoidal ground profile for the three cases $k > l_s$, $l_s > k > l_f$ and $k < l_f$.

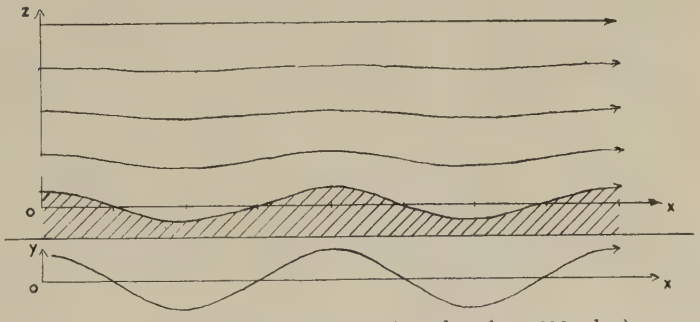
For $k > l_s$, see Fig. 1(a), the order of the wavelength of the ground profile, with typical atmospheric values, is up to about 2π km. We see that on this scale, the solution gives wavy streamlines whose amplitude decreases exponentially upwards. The critical wavelength up to which this solution applies is determined by the value of l_s, i.e., by the stability and speed of the airstream. In the atmosphere $l_s \gg l_f$ and on this scale l_f may be ignored, which is equivalent to saying that geostrophic forces have no opportunity to affect the result. There is thus negligible horizontal deformation in this case. The restoring force which makes the wave motion in the vertical possible arises solely from the stratification of stability, represented by l_s.



(a) $k > l_s$; small scale corrugation (wavelength $< 2\pi$ km).



(b) $l_s > k > l_f$; medium scale corrugation (wavelength 2π to $200\ \pi$ km).



(c) $k < l_f$; large scale corrugation (wavelength $> 200\ \pi$ km).

Figure 1. Streamlines produced by an infinite sinusoidal ground profile (after Queney).

For $l_s > k > l_f$, see Fig. 1(b), the order of the wavelength of the ground profile is in the approximate range 2π to 200π km. On this scale geostrophic forces as well as stability play a part and there is both vertical and horizontal deformation of the streamlines. The vertical deformation is not now subject to an exponential decrease upwards; the amplitude remains nearly constant but there is a retardation of phase with increasing height.

For $k < l_f$, see Fig. 1(c), the order of the wavelength is $> 200\pi$ km. and the scale is

such that geostrophic forces predominate in determining the deformation which is therefore almost entirely horizontal. Queney studied this further and found it necessary to introduce on this scale the latitudinal variation of the Coriolis parameter but he has not published a full account of this study.

(c) *Generalization of Queney's results*

As the differential equation of the wave is linear we are at liberty to superpose solutions to achieve a practical result. For a realistic two-dimensional mountain ridge, Queney used the profile represented by the Fourier integral

$$\begin{aligned}\zeta &= ab\int_0^\infty e^{ikx-ka}\,dk \\ &= ab\int_0^\infty e^{-ka}\cos kx\,dk \\ &= a^2 b/(a^2+x^2) \qquad . \quad . \quad . \quad . \quad . \quad (6)\end{aligned}$$

This is a synthesis of sinusoidal ground corrugations for all values of the wave number k, with the exponential factor e^{-ka} applied to each of them. The result is a single corrugation of bell-shaped cross-section. We see that its height is b and the half width is a, (the half width is the value of x, for which the height is half the maximum). We must ensure that $b \ll a$ otherwise the assumption that the disturbance quantities are small relative to their undisturbed values would be invalidated.

The displacement of the streamlines for flow over the ridge represented by Eq. (6) will be given by

$$\zeta = ab\sqrt{\frac{\rho_0}{\rho}}\int_0^\infty \exp\left[\, i\left(k\,x + k\left(\frac{l_s^2-k^2}{k^2-l_f^2}\right)^{\frac{1}{2}} z\right) - ka\right] dk \qquad . \qquad . \qquad (7)$$

The problem therefore reduces to the evaluation of this integral for the various cases with which it is desired to deal. The integral cannot be evaluated exactly in the general case but approximate solutions can be obtained. Queney found that the nature of the result depended very much on the scale of the ground deformation, represented by a, in relation to the critical values $1/l_s$ and $1/l_f$. The most significant of his results can be summarized as follows.

(i) $a \ll 1/l_s$.

This corresponds to the flow over a very small hill of width up to about 100 m. The effects of stability and the earth's rotation (represented by l_s and l_f in the integral) both prove to be negligible and the streamlines are given by

$$\begin{aligned}\zeta &= ab\left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}}\int_0^\infty \exp\{\, ikx - kz - ka\,\}\,dk \qquad . \qquad . \qquad (8) \\ &= ab\left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}}\frac{a+z}{(a+z)^2+x^2}\end{aligned}$$

The streamlines are given in Fig. 2 and it is seen that there is a single symmetrical wave over the hill. The displacement decreases both vertically and horizontally away from the hill and there is no train of lee waves.

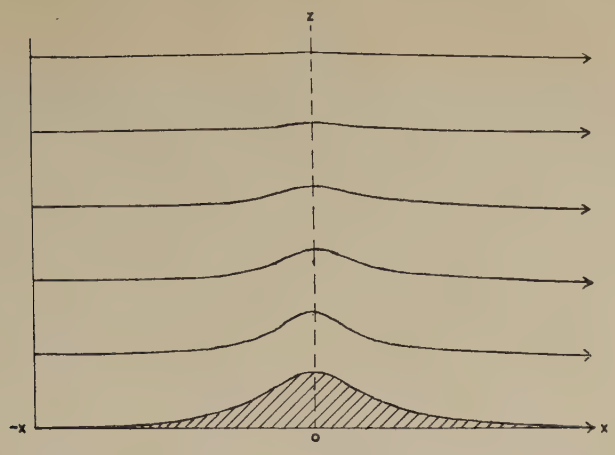


Figure 2. The flow over a very small ridge of width up to about 100 m (after Queney).

(ii) $1/l_s \ll a \ll 1/l_f$.

In this case the width of the ground obstruction is of the order of a few km. Queney found that on this scale the effect of the earth's rotation was still negligible but that the stability became important. The form of the integral applicable in this case is

$$\zeta = ab\left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} \int_0^\infty \exp\{ikx + il_s z - ka\}\, dk$$

$$= ab\left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} \frac{a\cos l_s z - x \sin l_s z}{a^2 + x^2} \quad . \quad . \quad . \quad (9)$$

The streamlines are illustrated in Fig. 3. There is a single wave above the hill having sinusoidal variation in the vertical, but no lee wave train. The important characteristic is that, with the entry of stability into the result (represented by l_s), the airstream is disturbed throughout its depth as distinct from the last case in which the disturbance decreased rapidly upwards. Geostrophic forces play no part and, accordingly, there is no horizontal deformation of the streamlines.

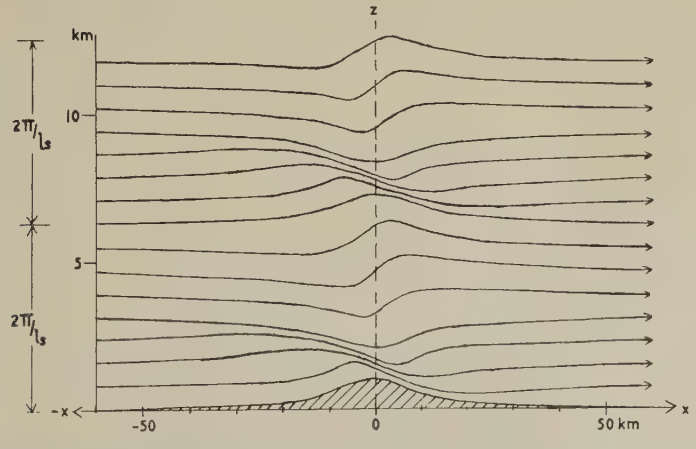


Figure 3. The flow of a uniform airstream over a ridge having a width up to a few km (after Queney). In the case illustrated, $l_s = 1$ km^{-1}, $a = 10$ km, $U = 10$ m/sec.

(iii) $1/l_s \ll a \simeq 1/l_f$.

This is the case of a still broader ground obstruction whose width is of the orde of 100 km. Both stability and geostrophic forces are significant on this scale and w have

$$\zeta = ab\sqrt{\frac{\rho_0}{\rho}}\int_0^\infty \exp\left\{ i\left(kx + k\sqrt{\frac{l_s^2 - k^2}{k^2 - l_f^2}}\, z\right) - ka\right\} dk \quad . \quad . \quad (10$$

After a transformation of co-ordinates Queney derived an approximate value fo this integral in terms of Bessel functions and the associated Struve functions. His streamı lines for this case are given in Fig. 4. There is a lee-wave system with horizontal as wel as vertical displacement of the streamlines. The horizontal disturbance is much large than that in the vertical and the horizontal wavelength is of the order of several hundrec km. Actually the horizontal wave length tends to $2\pi/l_f$ for large x. As regards th vertical deformation of the streamlines, there is a retardation of phase with height, th vertical wavelength being $2\pi/l_s$, i.e., dependent on stability. The lee waves increase ii amplitude upwards and persist further downstream with greater height. In many respects therefore, these "Queney" waves are unlike those responsible for ordinary lenticula clouds and exploited by glider pilots. We shall return to this later.

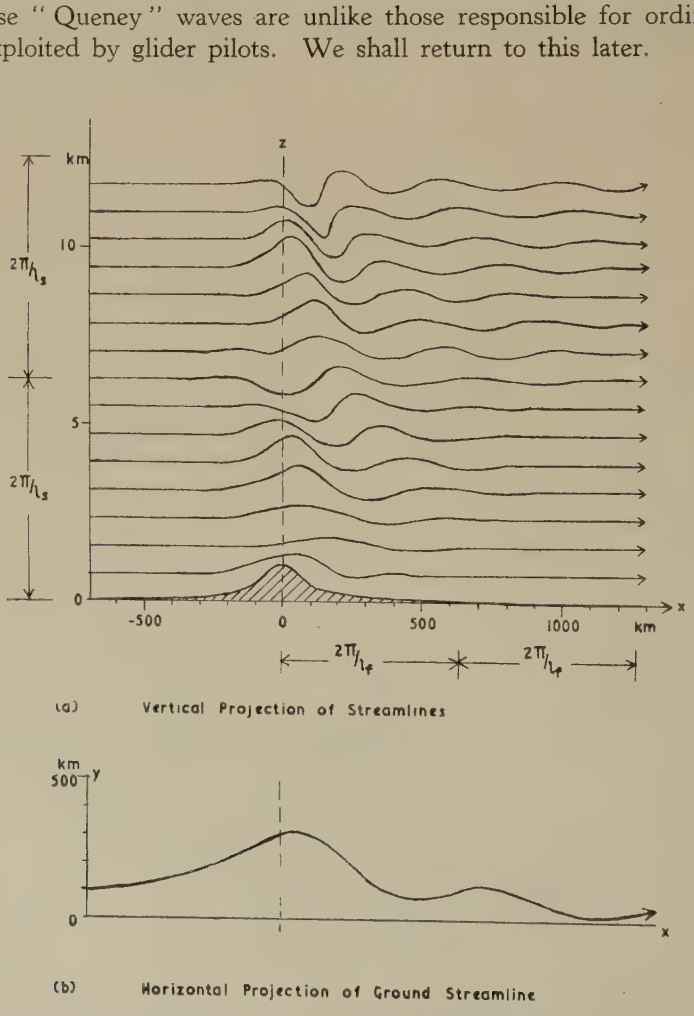


Figure 4. The flow of a uniform airstream over a ridge having a width of about 100 km (after Queney). In the case illustrated $l_s = 1\ \text{km}^{-1}$, $a = 100$ km, $l_f = \cdot 01\ \text{km}^{-1}$, $U = 10$ m/sec.

(iv) Queney also studied the case of a westerly airstream blowing across a very broad mountain range having a width of the order of 1,000 km, i.e. the case of $a \gg 1/l_f$. On this scale the variation of the Coriolis parameter with latitude also figures in the result and a system of stationary lee waves, mainly horizontal, was obtained, the wavelength being several thousand km. These waves have the nature of stationary Rossby waves and although of great interest from the synoptic view-point need not be discussed further in this review.

(d) *The work of Lyra (1940, 1943)*

This author published theoretical studies in 1940 and 1943 which at that time appeared highly significant and suggestive. Like Queney, Lyra confined himself to the uniform airstream case and employed the perturbation method, but for the profile of his elementary mountain ridge he used a narrow rectangular section developed from the Fourier integral $\int_0^\infty \cos kx\, dk$; the resulting profile has steep vertical sides. He chose the rectangular elementary obstacle with a view to obtaining the flow over any actual mountain shape by superposing the solutions for a number of rectangular sections selected so that when added they approximated to the desired mountain. In his solutions he obtained a type of lee-wave system in which the wave amplitude increases with height and decreases downstream; thus Lyra's lee waves are observable for a greater distance downstream the greater the height. Queney (and Scorer) used the mountain profile $\zeta = a^2 b/(a^2 + x^2)$, which with $b \ll a$ has a smooth shallow section with gently sloping and symmetrical sides. It appears from the work of both latter writers that Lyra's train of lee waves arises due to the steepness of the sides of the mountain section used by him. If a smooth shallow section replaces the rectangular shape, Lyra's train of lee waves disappears from the solution, which reduces to that derived by Queney and illustrated in Fig. 3, i.e., there remains a disturbance above the hill but no lee waves. Scorer (1953a) has discussed the way in which this occurs in the mathematical treatment. Thus although Lyra's theory was a contribution of considerable importance when published, it will not be discussed further here in view of the doubt as to the relevance of his lee waves in the practical problem of mountain airflow.

(e) *The work of Scorer (1949, 1951, 1952, 1953a, 1953b and 1954)*

Scorer recognized the inadequacies of the solutions obtained by Queney and Lyra. In both, the waves are almost negligible at low levels and die away rapidly downstream. In addition, Lyra's lee waves are somewhat artificial in that they depend on the choice of an unrealistic ground profile, whilst Queney's waves given in Fig. 4 are of very long wavelength and are mainly a horizontal oscillation about the geostrophic wind. Scorer realized that the observational evidence of mountain-wave clouds and the experiences of glider pilots, etc., suggested lee waves of a different character, viz., waves which increased in amplitude upwards to some level and then decreased to die away at great height, which had a wavelength of a few km and which in the absence of friction would continue indefinitely downstream over level ground to the lee of the obstacle.

Scorer considered frictionless, steady, laminar and isentropic flow but set out to provide for variations of lapse rate and wind shear in the vertical. He derives a second-order partial differential equation for the stream function of the disturbance and simplifies this on similar grounds to Queney. The earth's rotation is then neglected, as his subsequent study relates to small wavelengths (up to 50 km say). The simplified equation is

$$\frac{\partial^2 \psi}{\partial z^2} - \left(\frac{g}{c^2} + \beta\right)\frac{\partial \psi}{\partial z} + \left(\frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2} - k^2\right)\psi = 0 \quad . \quad . \quad (11)$$

The notation is as used for Eq. (1) applicable to Queney's work and we see, in fact, that the equation is the same except that the earth's rotation term in f does not appear, owing to the smaller scale of the problem to be studied, and there is the additional term in $\partial^2 U/\partial z^2$ representing the vertical rate of change of shear (U now varies with z).

Scorer neglects the second term in $\partial\psi/\partial z$ on the grounds that its effect is small and quantitative only. In the expressions for the displacement of the streamlines subsequently obtained, the result of this approximation is that the factor $\sqrt{\frac{\rho_0}{\rho}}$ which figures in Queney's results does not appear. The simplified equation used is thus

$$\partial^2\psi/\partial z^2 + (l^2 - k^2)\,\psi = 0 \quad . \quad . \quad . \quad . \quad (12)$$

$$\text{where } l^2 = \frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2}$$

Considering first an airstream in which l^2 is constant, we may use solutions of Eq. (12) for constant coefficients. The restriction $l^2 =$ constant does not imply the uniform airstream considered by Queney and Lyra since both β and U can vary with height within this limitation. The vertical displacement will be given by

$$\zeta = \frac{\psi}{U} = \frac{1}{U}(A\,e^{\mu z} + B\,e^{-\mu z}) \quad . \quad . \quad . \quad (13)$$

$$\text{where } \mu = +\sqrt{k^2 - l^2}$$

Once again we have the two cases :

(i) If μ is real, i.e., if $k > l$, we must have $A = 0$ otherwise the disturbance increases indefinitely upwards.

(ii) If μ is imaginary, i.e., if $k < l$ then

$$\zeta = \frac{1}{U}(A\,e^{i\nu z} + B\,e^{-i\nu z})$$

$$\text{where } \nu = +\sqrt{l^2 - k^2}.$$

We have again the difficulty that both components $e^{i\nu z}$ and $e^{-i\nu z}$ are indeterminate as $z \to \infty$. Scorer does not accept the application of the Rayleigh method of limits in this case and rejects Queney's (and Lyra's) choice of the downward component $e^{i\nu z}$. He argues instead that since the source of disturbance in the present context is at the ground, and there is no relevant energy source at high levels, the only realistic choice is the upward progressing wave. He therefore uses the solution :

$$\zeta = \frac{B}{U}e^{-i\nu z}.$$

The controversial difficulty as to the correct upper boundary condition has many complex ramifications and it is easy to exaggerate its importance in the practical problem. Those who wish to pursue the matter further are referred to a recent discussion by Scorer (1954) of the merits of the various devices which may be invoked to render the problem determinate.

For an infinite sinusoidal ground profile of wavelength $2\pi/k$, the streamlines of Scorer's elementary solutions are thus :

(i) $k > l$: $\zeta = \frac{U_0}{U_z} e^{ikx - \mu z} = \frac{U_0}{U_z} e^{-\mu z} \cos kx$ where $\mu = + \sqrt{k^2 - l^2}$

(ii) $k < l$: $\zeta = \frac{U_0}{U_z} e^{i(kx - \nu z)} = \frac{U_0}{U_z} \cos (kx - \nu z)$ where $\nu = + \sqrt{l^2 - k^2}$

These solutions are similar to those of Queney illustrated in Figs. 1 (a) and 1 (b) except that there is no horizontal deformation, the effect of the earth's rotation being neglected, and in the case of imaginary μ Scorer adopts the wave progressing upwards relative to the moving airstream, i.e. the slope of the loci of the crests is reversed.

(*f*) *Generalization of Scorer's results*

Scorer uses the ground profile represented by the Fourier Integral

$$\zeta = ab \int_0^\infty e^{ikx - ka}\, dk = a^2 b/(a^2 + x^2)$$

For an airstream in which l^2 is constant, the streamlines for the perpendicular flow across this ridge are given by

$$\zeta = \frac{U_0}{U_z} ab \int_0^\infty \exp (ikx - \mu z - ka)\, dk \quad . \quad . \quad (14)$$

If $l = 0$, the integral can readily be evaluated to obtain

$$\zeta = \frac{ab\,(a + z)}{(a + z)^2 + x^2} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

This solution applies to an airstream having neither static stability nor shear (i.e. $\beta = 0$, U = constant) for this would be a particular case of $l = 0$. The flow is seen to be similar to that obtained by Queney for a very small obstacle and illustrated in Fig. 2. We have therefore the interesting result that the flow over a very small hillock or obstacle up to about 100 m in width is dynamically similar to that of an airstream with no stability in the full-scale mountain problem. As we shall see later this is the only case in which wind-tunnel experiments might be carried out easily and with a good prospect of success. Scorer describes this type of flow as ' aerodynamic ' since gravity, stability and the earth's rotation play no part.

In order to emphasize the very important role played by stability, Scorer contrasts ' aerodynamic ' flow with the results obtained for an airstream with constant stability (or more precisely with constant l). The approximate value obtained for the integral of Eq. (14) becomes

$$\zeta = \frac{U_0}{U_z} ab \frac{a \cos lz + x \sin lz}{(a^2 + x^2)} \quad . \quad . \quad . \quad (16)$$

This is the same as Queney's solution for $1/l_s \ll a \ll 1/l_f$ except that the sign before the term $x \sin lz$ is reversed. This reversal occurs because of the different interpretation of the upper boundary condition; the streamlines have the same shape but their direction is reversed. However, as stated before, the important thing to note is that with the introduction of stability there is a complete change in the flow – the airstream is now disturbed throughout its whole depth. Scorer calls this ' barostromatic ' flow because of the vital part played by stability, i.e., by the stratification of potential density. It is to be noted that so far no trains of lee waves have appeared in these solutions of Scorer's, and this is now examined further.

The vertical velocity $w = \partial\psi/\partial x = ik\psi$, and thus we may write the wave equation (12) as follows

$$\frac{\partial^2 w}{\partial z^2} + (l^2 - k^2)\, w = 0 \qquad (17)$$

Considering the application of this equation to flow over level ground to the lee of an obstacle, we require that $w = 0$ at the ground and $w \to 0$ as $z \to \infty$. It is clear that if these boundary conditions are imposed with constant l, w will be everywhere zero and therefore we can have no lee waves with such an airstream. For realistic lee waves we must have w finite at some level(s) and also zero at $z = 0$ and tending to zero as z becomes very large. Thus $\partial^2 w/\partial z^2$ must change sign at least once; in fact it can be inferred that we must have l^2 less in some fairly deep upper layer than in a layer below. This notion led Scorer to introduce, for purposes of theoretical study, the concept of an interface in the airstream, dividing it into two layers in each of which l^2 has a constant but different value. There is then a different solution of the wave equation for each layer, corresponding to the two different values of μ, and accordingly four arbitrary constants. For the four boundary conditions required we have those at the ground and at infinite height, whilst continuity considerations at the interface supply the remainder.

If for such an airstream, suffixes 1 and 2 refer to the lower and upper layers respectively, and for convenience we take the interface at $z = 0$ and the ground at $z = -h$, then the vertical displacement will be given by

$$\text{Upper layer } (z > 0): \qquad \zeta_2(z) = \frac{A}{U_z} e^{-\mu_2 z} \qquad (18)$$

$$\text{Lower layer } (0 > z > -h): \qquad \zeta_1(z) = \frac{A}{U_z} (\cosh \mu_1 z - \frac{\mu_2}{\mu_1} \sinh \mu_1 z) \qquad (19)$$

Here the boundary conditions at the interface and at infinite height have already been introduced whilst A awaits the ground boundary condition. For the flow over level ground to the lee of the (so far) unspecified disturbance of wave number k, waves may only occur if $\zeta_1(-h) = 0$, i.e., if

$$\mu_1 \cosh \mu_1 h + \mu_2 \sinh \mu_1 h = 0 \qquad (20)$$

As $\mu_1 = \sqrt{k^2 - l_1^2}$ and $\mu_2 = \sqrt{k^2 - l_2^2}$, Scorer regards Eq. (20) as an equation in k determining the wave number of those waves which could persist in the airstream over level ground when once initiated by a disturbance upwind. He shows that Eq. (20) has no solution unless $l_1^2 - l_2^2 > \pi^2/4h^2$, whilst if $l_1^2 - l_2^2 > 9\pi^2/4h^2$ there will be two solutions, for $l_1^2 - l_2^2 > 25\pi^2/4h^2$, three solutions, and so on. However, it is also shown that the lee waves corresponding to the smallest root, denoted by k_0, will be amplified to a much greater extent than the others which in practice may be ignored.

Allocating the value of A for flow over the sinusoidal ground profile e^{ikx} (real part) the streamlines in the lower layer become

$$\zeta(z) = \frac{e^{ikx} f_z(k)}{f_{-h}(k)}$$

$$\text{where } f_z(k) = \frac{1}{U_z} (\cosh \mu_1 z - \frac{\mu_2}{\mu_1} \sinh \mu_1 z)$$

Generalizing, we have that the streamlines for the flow over the ridge represented by

$$\zeta(-h) = ab \int_0^\infty e^{ikx - ka}\, dk = a^2 b/(a^2 + x^2)$$

will be given by

$$\zeta(z) = ab \int_0^\infty \frac{f_z(k)}{f_{-h}(k)}\, e^{ikx - ka}\, dk$$

As $f_{-h}(k_0) = 0$, the integrand is infinite for $k = k_0$. Scorer invokes contour-integration methods to derive as a close approximation for the streamlines in the lower layer :

$$\zeta(z) = ab \frac{f_z(0)}{f_{-h}(0)} \frac{(a + ix)}{(a^2 + x^2)} + 2i\pi ab \frac{f_z(k_0)}{\frac{\partial f_{-h}}{\partial k_0}(k_0)} \quad \text{for } x > 0 \quad . \quad . \quad (21)$$

$$\zeta(z) = ab \frac{f_z(0)}{f_{-h}(0)} \frac{(a + ix)}{(a^2 + x^2)} \quad \text{for } x < 0 \quad . \quad . \quad . \quad . \quad (22)$$

In the upper layer the lee waves, represented by the second term in the expression for $x > 0$, die away exponentially being proportional to $e^{-\mu_2 z}$.

Scorer has exploited this result for a simple two layer model to calculate the streamlines in selected cases. One of his examples is reproduced as Fig. 5. Since $l^2 = g\beta/U^2 - 1/U\, \partial^2 U/\partial z^2$, there is considerable latitude in the choice of the wind and temperature profiles whilst still ensuring that l^2 is constant within each of the two layers. In fact in the example illustrated, there are effectively three layers but the value of l^2 in two of them is the same. The characteristics of the wind and temperature profiles used by Scorer for this example are shown on the left of Fig. 5; they are similar to those commonly found in nature when waves are observed, viz., marked forward wind shear through a stable layer at fairly low levels. The lee waves predicted by the theory also accord well with what is commonly observed. By avoiding the artificiality of the uniform airstream case studied by Lyra and Queney, and dealing with more realistic model airstreams, Scorer has revealed the significance of variations of lapse rate and wind speed in the development of lee waves.

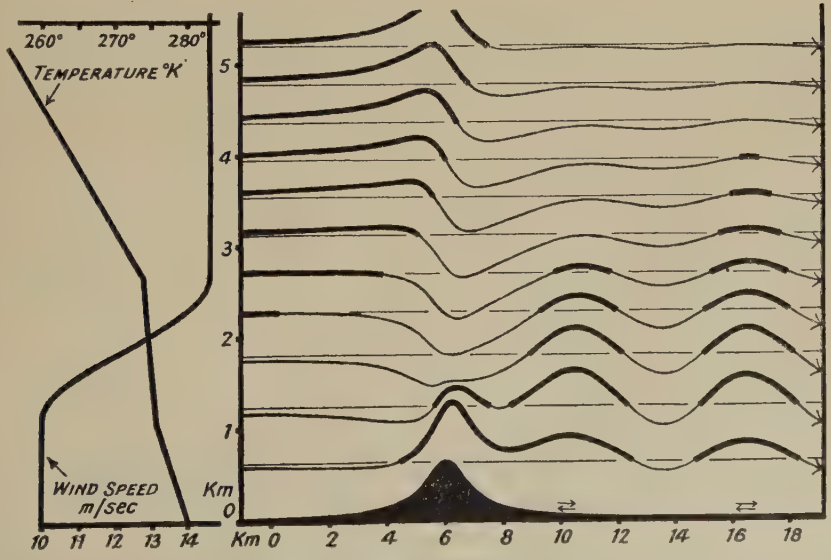


Figure 5. Airflow over a mountain ridge under conditions favourable for the occurrence of lee waves (after Scorer).

Multilayer models, with several different values of l^2 could in theory be set up and examined in similar fashion. However, the mathematics would become inordinately complex and no new general principle could emerge. In practice where l^2 may vary continuously it would remain true that for lee waves to occur l^2 must assume lower values through some fairly deep upper layer than in some layer below (but see qualifying comments below on inversions).

We have led up to this result in some detail because it is of fundamental importance and it seems likely that it may prove useful for practical forecasting. Scorer has, however, deduced a number of other conclusions which it is now proposed to summarize without giving the derivation in full detail.

(1) *Two-layer models.* (i) In the case of the simple 2-layer model, Eq. (20), which determines the wavelength, only has solutions if $l_1^2 - l_2^2 > \pi^2/4h^2$. Thus the magnitude of the decrease in l^2 must exceed a limit depending on the depth of the lower layer.

If $l_1^2 - l_2^2 > 9\pi^2/4h^2$ one of the two possible lee waves will have within the lower layer a nodal surface where the wave has zero amplitude and the phases above and below are opposite.

The properties of the possible lee waves for $l_1^2 - l_2^2 > 25\pi^2/4h^2$, etc., can be deduced similarly.

(ii) Distinguishing for a moment between the disturbance over the hill itself (the so-called hill-lift) and the lee-wave train, if any, the hill-lift disturbance will be greater and will extend to greater heights if l^2 decreases upwards, i.e., the condition for a large disturbance over the hill itself is the same as for lee waves.

(iii) We have noted that the amplitude of the lee waves increases upwards to some level above which it decreases to die out in the upper levels. For a given simple two-layer model the level of maximum amplitude can be obtained from Eq. (21) but in the general case we may say that the maximum amplitude will occur roughly where l^2 is greatest.

(2) *Adiabatic layers.* (i) In the atmosphere adiabatic layers in the lower levels are common. In such layers $l^2 = 0$ (neglecting shear) and it might appear that Scorer's condition for lee waves could not be satisfied. However, the condition for lee waves is that l^2 should be greater in *some* lower layer than in the layers above; the largest values of l^2 do not necessarily have to occur in the surface layer. Scorer has studied this further and finds that waves are still possible with an adiabatic ground layer, provided his criterion is satisfied above, but the magnitude of the disturbance aloft will then be less than with stable stratification down to the surface. He has in fact published the streamlines calculated for a three-layer model in which the bottom layer is adiabatic (Scorer 1953a, Fig. 7) and obtained lee waves in such a case.

(ii) If there is an adiabatic layer (or more generally a layer in which $l^2 = 0$) in middle levels, and one starts from the assumption that steady laminar flow with lee waves is possible, it is found that the boundary conditions at the top and bottom of the adiabatic layer are related. Since these conditions are necessarily independent it is concluded that the kind of flow assumed cannot occur, i.e., that steady laminar flow over a mountain must break down if there is an adiabatic layer in middle levels. Perturbation theory cannot assist in determining the type of flow in such a case.

(3) *Inversions.* (i) The development of stationary lee waves on flowing water is closely connected with the large density discontinuity at the surface. Several writers, notably Küttner (1939), have recognized that an inversion in the atmosphere might possibly act in the same way and thus have a bearing on the occurrence of atmospheric lee waves. However, the work of Queney and Scorer indicates that one need not introduce an inversion in the model airstream considered in order to obtain lee waves. Scorer, in fact, has been at great pains to emphasize in many papers that an inversion is not a dynamical requirement

for lee waves. It is true that an inversion is commonly found in airstreams in which waves are observed, but as has been pointed out an inversion would commonly be associated with a distribution of humidity favouring wave clouds, i.e., conditions in which any waves become visibly revealed. It seems, however, that although not a dynamical necessity for waves, an inversion could make them possible when the criterion regarding the variation of l^2 with height does not appear to be otherwise fulfilled. An ideal inversion would be a layer of no depth and infinite l^2, but in practice we can regard an inversion as a very shallow layer through which the value of l^2 is very large. The occurrence of waves in airstreams containing inversions is therefore quite consistent with the general criteria regarding the variation of l^2 with height. This has been confirmed in a recent study (Scorer 1954) of the effect of inversions and shallow stable layers of various intensities on the occurrence and amplitude of lee waves.

(4) *Diurnal variation.* In the lower layers of the atmosphere, the lapse rate, wind speed and wind shear (and therefore the value of l^2) are subject to large diurnal variations especially near sunrise and sunset. Generally speaking the net result is a reduction in the value of l^2 after sunrise and an increase around dusk. In theory this would mean an increase after sunrise in the wavelength of any waves until perhaps the waves ceased owing to infringement of the condition $l_1^2 - l_2^2 > \pi^2/4h^2$, whilst around dusk the waves might reform if and when the condition was again satisfied. Both these effects have been observed and reported by other writers, e.g. Manley (1945) and Roper (1952). It is appropriate to mention here that on similar grounds, one would expect a greater frequency of wave effects in winter than in summer; this too is in accord with observation.

(5) *Oblique flow and other mountain shapes.* So far discussion has been limited to the airflow perpendicularly across an infinite symmetrical ridge. This lends itself to theoretical study because the problem is then essentially only two-dimensional. In theory one can deal with more complicated two-dimensional profiles by superposing the solutions for different ridges. In this way the streamlines for the flow across an asymmetrical ridge and various plateaux have been obtained.

For oblique flow across a ridge, the component along the ridge remains undisturbed whilst the component across the ridge is disturbed as in the case of perpendicular flow. Scorer (1953a) has shown that if the undisturbed stream contains a component of shear parallel to the ridge, the total shear in the disturbed stream may assume almost any direction. This conclusion has a bearing on the formation of billow clouds and also may be significant in connection with the navigation of aircraft over mountain ranges.

Theoretical study of the flow over an isolated hill as distinct from an infinite ridge is complicated since the problem becomes fully three-dimensional. One can exploit the result for oblique flow over a ridge, by synthesizing the flow patterns for an infinity of ridges whose axes are inclined in all horizontal directions through a point. The mathematical treatment for this approach to the three-dimensional problem has been indicated by Scorer (1954) who has been able to obtain in this way the distribution of vertical displacement for the flow over a single hill (Scorer 1952). These latter results are consistent with the bow-shaped orographic cloud which has been observed in such a case and discussed by Abe (1932). As regards the development of waves to the lee of a single hill, the general conclusions are that any waves will be of smaller amplitude than those to the lee of an infinite ridge of similar section and that the waves will decrease in amplitude rapidly downstream. This is in contrast to the infinite ridge case when the lee waves (in the absence of viscosity) persist indefinitely downstream. For a ridge of finite length lying across the airstream the results are intermediate, viz., the persistence of any lee waves downstream is greater the greater the length of the ridge.

(6) *A layer of calm air.* If an airstream has a layer of calm air between some levels, the difficulties about the flow at great heights, and in particular about the boundary condition at infinite height, disappear. Scorer studied such a case and found that lee waves, confined to the airstream below the calm layer, were possible provided certain conditions were satisfied. This case appeared to be of academic interest only, but recently a case of lee waves has been reported (see Scorer 1953b) in circumstances which were possibly of this type.

(g) *The validity and limitations of the results of perturbation theory*

In order to assess the practical value of the theoretical results deduced from the applications of the perturbation method, it is necessary to review the initial assumptions. The main assumptions involved are commented on below.

(i) *Viscosity.* The effect of the assumption of frictionless flow was examined by Queney, who concluded that viscosity would merely exert a damping effect on the disturbances. The damping effect would be small and would not alter the results qualitatively.

(ii) *Isentropic flow.* The assumption that the potential temperature is constant along the streamlines involves neglect of radiation and condensation effects. Radiative effects would in general be too slow to affect the results on the scale considered by Scorer, but on the synoptic scale (e.g., for the long waves of the stationary Rossby type) they would probably be important. However, the latter question has not been fully studied and is irrelevant in the present review.

As regards condensation, the results could easily be applied to a completely cloudy airstream by using wet-bulb potential temperature in the coefficient of stability β. One could similarly allow for an 8/8 layer of cloud but not readily for broken clouds. In the case of an airstream containing, say, well-broken cumulus, most of the atmosphere (i.e., that between the clouds) would be stably stratified and the conclusions from perturbation theory would be closely applicable. This means that convection is not necessarily inconsistent with the simultaneous occurrence of orographic waves, although it must at least complicate the flow to some extent by mutilating the laminar flow.

(iii) *Steady laminar flow.* The restriction to steady laminar flow is serious because there is ample evidence that the flow over mountains is commonly otherwise. No mathematical treatment of unsteady or non-laminar flow has yet been put forward and it is unlikely that much progress could be made by such a theoretical study until some completely new and superior mathematical approach is forthcoming. In the meantime, we must be content with what has been learnt of the flow in such cases from observational evidence. Here the work of Förchtgott, although in many respects inconclusive, is valuable and illuminating and it is probable that extended field studies on similar lines would be profitable. (Förchtgott's work is discussed in Section 5 of this review.)

(iv) *Displacements small.* The displacements, and indeed all the disturbance quantities, are taken to be small compared with the corresponding undisturbed values. This enables the equations to be linearized since the products and squares of the disturbance quantities are then justifiably neglected. The condition is satisfactorily fulfilled if the height of the mountain is small compared with its width. It is reasonable to accept the results as qualitatively correct (and very nearly quantitatively also) provided the mountain height is no more than about ten per cent of the width, although Queney claims rough validity if the ratio 'height to width' is up to $\frac{1}{4}$ or even $\frac{1}{2}$ in special cases. We must bear in mind, however, that with increasing steepness of the mountain ridge, separation of the flow from the surface becomes increasingly probable. This would be manifest in

the form of lee eddies or some other form of motion and would constitute partial violation of the assumption of laminar flow. The presence of a lee eddy can be regarded as changing the effective shape of the mountain and thus of the lowest streamline; above this the flow may remain laminar.

(v) *The earth's rotation.* For Queney's waves, the earth's rotation provides one of the essential restoring forces, the scale of the phenomena considered being large enough for this to be possible. For Scorer's lee waves, the scale is too small for the earth's rotation to have any significant effect since he confines his study to cases when the mountain ridge and the regime considered are traversed by the airstream in an hour or two at the most. With this restriction, neglect of the earth's rotation in Scorer's examples is quite justified, but his results would not be applicable to large-scale phenomena.

(*h*) *Long's treatment*

Long (1953a) has published a theoretical study dealing with the flow of stratified fluids. He was concerned at the time with producing a rigorous treatment of such flows in connection with his experimental work with models referred to in the next section. In his model, the fluid is stratified but incompressible and for the theoretical study is taken to be flowing between upper and lower rigid boundaries. One of the many interesting features of Long's study is that he has obtained analytical solutions for finite amplitude disturbances, albeit for one special case. In the theoretical studies of the atmospheric problem already discussed, the authors used perturbation theory at the outset in order to make possible the linearization of the equations. Long derived a general differential equation for the variation of a disturbance without restricting himself to small perturbations. The equation is, of course, highly non-linear and intractable in the general case but Long discovered that for one special case it reduced to a simple linear form. The case is one in which the density gradient with height is linear and also $\rho\, U^2 = \text{const}$. The simplified equation for this case is effectively the same as the wave equation derived by Scorer for small perturbations, except that for the liquid model, density replaces potential temperature in the coefficient of stability.

The significance in the atmospheric problem of Long's solutions for finite amplitude waves is not yet fully clear. The doubt about the atmospheric analogy arises not because of any direct effect of the compressibility of the atmosphere, for it is known that the compressibility can be neglected (see Section 4). It arises because of the indirect effect of compressibility on the upper boundary. In the atmosphere there is no upper boundary, and this constitutes a fundamental difference from the upper boundary in Long's model. However, the attainment of solutions for finite waves is a significant step forward, whatever the other limitations of the solutions, and it is hoped that it will be possible to exploit this result further in the case of the atmosphere. Another possibility would be to examine the general equation for finite waves with a view to deriving solutions by numerical methods in cases other than the special one for which the equation reduces to linear form.

4. Model experiments

The hope that model experiments could be used to advance our understanding of the difficult problem of mountain airflow has been cherished by many writers on the subject in the past and a number of attempts have been made to exploit model techniques. These attempts have met with little or only partial success. In some cases no consideration at all was given to the need for achieving dynamical similarity between the model and the full-scale problem, whilst in other cases the only precaution taken was to obtain a

reasonable Reynolds number in the model on the assumption that molecular viscosity in the model would correspond to eddy viscosity on the full scale. In defence of these past attempts, however, it must be mentioned that the theoretical studies described in the last section were not available at the time most of the model experiments were carried out; furthermore no general study of the conditions for dynamical similarity of atmospheric motions, such as that published recently by Batchelor (1953), had then been made. It is probable, therefore, that the very great importance of, and the criteria for, dynamical similarity were not appreciated by model experimenters in the past.

The differential equation used by Scorer to determine the wave motion, in cases when the scale of the problem permits the earth's rotation to be neglected (Section 3, Eq. (12), notation as before), is :

$$\frac{\partial^2\psi}{\partial z^2} + \left(\frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2} - k^2\right)\psi = 0$$

Here $g\beta/U^2$ represents the effect of stability or the stratification of potential density and $1/U\ \partial^2U/\partial z^2$ represents the effect of the vertical rate of change of shear. k is the wave number having the units (length)$^{-1}$, and figures instead of a representative length. Scorer argues that if a model experiment is to reveal accurately the motion in the full-scale problem, the conditions in the model experiment must be so arranged that the relationship between the terms of the equation is the same both for model and full scale. That is, the non-dimensional number :

$$S = \frac{1}{k^2}\left(\frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2}\right) \quad . \quad . \quad . \quad . \quad (23)$$

must have the same value on both scales. We must bear in mind, however, that Eq. (12) is not exact, several assumptions and approximations being involved in its derivation. It is not, therefore, immediately apparent that the conditions for dynamical similarity can be deduced in such a manner from the approximate wave equation. The conditions so obtained depend on the assumptions and approximations being justified on the model, as well as on the full scale and a separate examination of the relative magnitude of the terms of the full equation as applicable to the model problem would seem to be first necessary. Alternatively, one could avoid this difficulty by deducing the conditions from the basic equations of motion, i.e., before linearization and approximation. If this is done, the rigorous conditions as derived by Batchelor (1953) are obtained. These conditions, however, reduce to equivalence of the ' Scorer number ' $S = g\beta/k^2U^2$ if certain limitations are imposed on the flow. The main limitation is that it must be possible to regard the atmosphere as behaving like an incompressible fluid. It is well known that potential temperature in the atmosphere behaves almost exactly like density in a stratified incompressible fluid (this can be appreciated from the relation $g/c^2 + \theta^{-1}\ \partial\theta/\partial z + \rho^{-1}\ \partial\rho/\partial z = 0$), so that the limitation in the present context is acceptable.

Assuming that the degree of similarity achieved by equating the ' Scorer number ' is acceptable for a particular experiment, the model flow-conditions can be calculated for a typical case. For a model 3×10^{-5} times full size, with $U = 10^3$ cm/sec and $\beta = 2 \times 10^{-7}$ cm^{-1} (a lapse rate of about 4°C/km) on the full scale, then even with the very large temperature gradient of 1°C/cm in the model, the wind-tunnel flow speed would have to be as low as 4 cm/sec. For faster flow speeds very much greater temperature gradients would be required. It is clear that Scorer regarded it as impracticable to obtain such experimental control and abandoned pursuit of the possibility of wind-tunnel experiments.

It is interesting to consider for which full-scale cases wind-tunnel experiments could legitimately be regarded as valid, if no control at all is exercised over the temperature gradient in the model. If in the wind tunnel the temperature and flow speed are constant, then the ' Scorer number ' for the model is zero. Thus the experiment should reproduce the full-scale flow only in the special case when $l^2 = 0$ on the full scale. Such experiments might, therefore, be of limited value if one wished to investigate the flow over hills in, say, northern India in the pre-monsoon period when the lapse rate is nearly dry adiabatic up to 10,000 ft or more, i.e., a case when the static stability is zero or very small for a considerable depth. In addition, and this might be more useful, the experiments could be applied to examine the flow of a completely cloudy airstream over a mountain for then the static stability (and thus l^2) would be approximately zero in the full-scale case as well as in the model. The type of case in mind for this is one in which the airstream contains 8/8 nimbostratus and has a wet adiabatic lapse rate throughout a great depth. Apart from the special full-scale case in which $l^2 = 0$ we may infer from the theory of the last section that a wind-tunnel experiment without control of temperature would reproduce the flow over a small obstacle or hill up to about 100 m in width since, as we saw, the full-scale flow in such a case is practically independent of the stability effect, i.e., of the variation of l^2.

If we wished to devise model experiments to investigate larger scale phenomena in which the earth's rotation is concerned, e.g., Queney waves, it would be necessary for the non-dimensional number

$$Q = f^2/k^2 U^2 \quad . \quad . \quad . \quad . \quad (24)$$

to be equated in both model and full scale. This condition is effectively the same as equality of the Rossby number and must be satisfied in addition to equality of the Scorer number. The condition follows from consideration of Eq. (1) and it would ensure the proper representation of geostrophic forces in the model experiment. In a typical case, the condition would require rotation of the model at about one revolution per 2/3 min to simulate the earth's rotation. It seems unnecessary to pursue this further here since mountain airflow on the large scale implied is not of direct concern to aviation.

In connection with the Gibraltar investigation of 1929-30, Field and Warden (1933) supplemented their field work by wind-tunnel experiments on a 1 in 5,000 model of the Rock. As they made no arrangements to simulate stability, the flow obtained in the model should have corresponded to the flow in the full-scale case of an airstream with zero static stability. In spite of this, what they described as remarkable agreement between the flow in the experiment and the flow on the site was obtained. We must remember, however, that the flow on the site was in no sense fully mapped out; it was only possible to determine some of the characteristics of the flow to a limited extent from the programme of balloon work. Furthermore they were dealing with a constantly changing turbulent flow pattern which made specific comparison between the model experiment and the site work much more difficult than would have been the case had the flow been steady and laminar. One of the main conclusions was that the Rock would often disturb the airflow up to 5,000-6,000 ft. However, cumulus initiated by the Rock commonly reaches to a similar level and this may have been at least partly responsible for the apparent agreement between site experience and model experiment. The distinction being drawn here is that between turbulence originating purely as a characteristic of the flow over an obstacle, as in a wind tunnel, and turbulence arising through convection set off by the lift over the obstacle.

Abe (1952) studied the mountain clouds around Mount Fuji in Japan from 1920 onwards and from time to time supplemented his site study with wind-tunnel experiments. He used a 1 in 50,000 model and selected a tunnel-flow speed so as to equate

the Reynolds number to that applicable on the full scale; this was, of course, on the usual assumption that molecular viscosity in the model corresponded to eddy viscosity on the full scale. The tunnel-flow speeds required for this were of the order of 2 mi hr^{-1}. In an attempt to achieve greater realism, Abe arranged wind shear and a temperature gradient in the model experiments. For wind shear, he introduced an extra blower or used partial interruption of the flow, and for a temperature gradient he introduced dry-ice smoke at the bottom of the tunnel. He also reproduced a change of wind direction with height by introducing blowers at the side of the tunnel. So far as is known, this has been the only attempt to simulate shear and stability in wind-tunnel experiments. Unfortunately Abe did not arrange these effects quantitatively to ensure that strict dynamical similarity was achieved, and we must assume that if there was complete similarity it was accidental. He photographed the model flow and found approximate qualitative agreement with some properties of the flow on the site, in so far as he had deduced them from observations of the mountain clouds. In particular in both cases many vortices were observed with vertical, horizontal or inclined axes. Abe's work tells us a good deal about the characteristics of the flow over and around Mount Fuji. It assists very little towards a more complete understanding of the problem of mountain airflow in general.

Rouse (1951) has discussed the potentialities of model experiments and refers, without giving details, to the value of experiments on large models of mountains in hangars using blowers or airscrews to provide the flow. We have seen that a model experiment in a wind tunnel without control of stability will with certain restrictions give dynamical similarity with cases of the flow over obstacles and small hills up to about 100 m in width (and in addition with a full-scale mountain problem in the special case of $l^2 = 0$). To proceed, therefore, from the wind-tunnel scale to the scale of hangar experiments does not avoid the awkward problem of controlling the temperature gradient to simulate stability. Some advantage might derive from the increase of scale, however, in so far as much smaller temperature gradients would be required on the hangar scale than in the case of the wind tunnel.

So far, we have not discussed viscosity. If viscosity is introduced into the equations of motion by means of the usual terms, it is found that for dynamical similarity the Reynolds number must be equated on both scales in addition to the other dimensionless numbers already discussed. It has usually been suggested that if this equality is achieved by using molecular viscosity in the model and eddy viscosity in the full scale, the simulation of this feature will be satisfactory. On the face of it, this idea seems of dubious validity, and although doubt has been expressed by some writers, it does not appear to have been thoroughly examined in the literature. Possibly the similarity hypotheses and other ideas of Kolmogoroff and Batchelor recently discussed by Batchelor (1950) would be of value here.

In the course of a study of the flow of stratified liquids, Long (1953a) has carried out experiments using two- and three-layer systems of immiscible fluids in a long glass-sided tank. In this case the obstacle, which filled the space between the front and back glass walls, was steadily drawn at various speeds along the bottom of the tank by mechanical means. The deformations of the three layers of fluid were studied and photographed. Long applied a rough dimensional analysis to the obvious parameters of his model arrangement and considered that his experimental conditions roughly reproduced the effect of stability and in fact suggested that the two interfaces might approximately correspond to an inversion at mid-tropospheric levels and the tropopause. The flow obtained in the three-layer system exhibited broadly some of the characteristics of the ' Bishop ' wave. In some of Long's photographs there is some sign of the fluid ascending towards the obstacle at low levels and descending at higher levels above the same position. This is interesting

since the streamlines calculated by perturbation theory for many atmospheric cases show the same feature and the validity of this aspect of the theoretical results has not in the past been readily accepted. The use of layers of liquid of different densities for the simulation of stability appears to have much to recommend it as, presumably, the adjustment of densities in such an arrangement poses a far simpler laboratory problem than the production of a very large temperature gradient in a wind tunnel. Admittedly the compressibility of the atmosphere is not reproduced, but as has been indicated its neglect is justified (ignoring in the present context the upper-boundary problems which compressibility introduces); the analogy between the stratification of density in a fluid model and that of potential temperature in the atmosphere is fully adequate.

In more recent, and as yet unpublished, work reported briefly to the O.S.T.I.V. Congress, Buxton, 1954, Long has succeeded in performing experiments with continuous density gradients, these being achieved in his model with salt solution. He has studied the disturbed flow patterns obtained with various values of the internal Froude number, a single value of which, with a linear density gradient, is sufficient to specify the characteristics of the basic flow in the model. The limitations of the atmospheric analogy arise from the upper-boundary difference between model and atmosphere and the fact that shear is so far not reproduced. However, Long's success in simulating stability, coupled with his theoretical solutions for finite amplitude disturbances (see Section 3 (h)), make his methods appear potentially powerful. The publication of a full account of this work is awaited with considerable interest.

5. The work of Förchtgott

Förchtgott (1949, 1952) has carried out intensive investigations of the airflow over ridges during the post-war years. He is, amongst other things, a gliding instructor who has specialized in soaring in mountainous areas and in view of his wide experience his ideas on the subject must command respect. His work has comprised a series of field studies at various mountain locations in Bohemia using light aircraft and groups of gliders fitted with instruments. The flights were supplemented by the study of mountain clouds (both visually and by time-lapse photography), of smoke and of the flight of birds. In all, during the period of 1946-50, he gathered about 35 different sets of observations spread over five different mountain locations. By studying the observational data gathered and drawing on his own considerable gliding experience, Förchtgott has built up a fairly comprehensive theory which is presumably consistent with both the data and his experience. His classification of mountain airflow into four main types is attractive and plausible but is deduced mainly by a process of subjective interpretation. It is not clear to what extent we can regard his conclusions as being of general validity.

Förchtgott has been severely critical of the conclusions deduced from perturbation theory on the grounds that by neglecting the turbulent layer and treating the flow as wholly laminar and steady, theorists have ignored a vital feature of mountain airflow. However, it must be stated that the significance of the restriction to steady laminar flow has been most fully acknowledged in all theoretical treatments, and in point of fact there is little or no conflict between Förchtgott's results and those of perturbation theory.

Förchtgott found that no organized flow patterns could be discerned unless the airstream had stable or neutral stratification, the wind was roughly normal to the mountain ridge and the wind direction varied little with height. Accordingly his studies were confined to situations when these conditions were satisfied. He found that mountain ridges with steep lee slopes were most favourable for lee wave effects and that the topography of the upwind slope had no evident importance. Apart from these provisions

he concluded that the type of flow depended on the speed of the wind as compared with the height of the ridge above the lee plain and the variation of wind with height. His classification on this basis into four main types of flow is given in Fig. 6, the wind profile which he found to be appropriate to each flow type being indicated at the left of each diagram. The flow types are now briefly commented upon.

(a) *Undisturbed streaming.* This occurs with very light winds; each streamline executes a single shallow wave and the vertical displacement of the streamlines dies out rapidly with height.

(b) *Standing eddy streaming.* Slightly stronger winds are required for this type. The surface streamline separates from the ground and there is a stationary lee eddy with approximately horizontal axis. The crest in the laminar streaming above the eddy is displaced downwind from the mountain, and we may regard the effective shape of the mountain as being changed by the presence of the eddy. Reverse flow up the lee slope is implied in this type of flow and has been observed frequently in mountain areas.

(c) *Wave streaming.* With still stronger winds, the stationary lee eddy disappears and a system of lee waves develops in association with a stationary vortex system having both clockwise and counter-clockwise rotations in two horizontal rows. Turbulence is found in the vicinity of the vortices. The waves die out both horizontally and vertically away from the ridge. Thus Förchtgott does not find in practice steady laminar waves streaming down to the surface on the lee side. He always finds some form of turbulent wake to the lee of the ridge even when there are smooth waves above. However the evidence for stationary closed vortices does not seem conclusive and we must remember that an impression of rotation of the bar clouds, or so-called ' rotors,' would be provided by the wind shear in the wave system. The turbulence too could be accounted for by shear, which in theory would be intensified in the crests and troughs of the waves.

(d) *Rotor streaming.* With very strong winds extending through a restricted vertical depth comparable with the height of the mountain (see wind profile in Figs. 6(d) and 6(e)), the double-vortex system becomes intensified and the spacing closes up towards the mountain. There is in this case no significant wave system above the vortices. Flight investigation of this type of flow was found very difficult owing to heavy turbulence, especially close to the lee slope. In discussing the conditions for this type of flow, Förchtgott introduces the term ' streaming layer ' and speaks of critical values of its depth. The streaming layer is not defined, but he appears to have in mind the layer in which there is a definite component across the ridge either increasing or remaining roughly constant with height. A strong component of reverse shear, reducing the component across the ridge to zero or small values at some level, is regarded as providing a streaming layer of limited depth. Förchtgott claims that wave streaming breaks down into rotor streaming when the depth of the streaming layer falls to about $1\frac{1}{2}$ times the height of the ridge. Further decrease in the depth of the streaming layer transforms the flow pattern into a completely chaotic type in which no organized features can be recognized.

As regards the limits of wind speed determining which of the types of flow (a), (b), (c) will appear in any given case, Förchtgott's observations indicated that these would depend upon the height of the ridge above the lee ground. He quotes, for example, that for a ridge 900 m high, a wind of 12 m/sec is necessary for wave streaming. If the obstacle is a solitary hill, rather than a very long ridge, a much stronger wind speed is necessary for a given type of flow than in the case of a long ridge of a similar height.

Förchtgott has assembled his various deductions in the form of a diagram from which, it is claimed, one could decide which type of flow to expect in any given case. The variables are the height of the hill, the wind speed and the depth of the streaming layer,

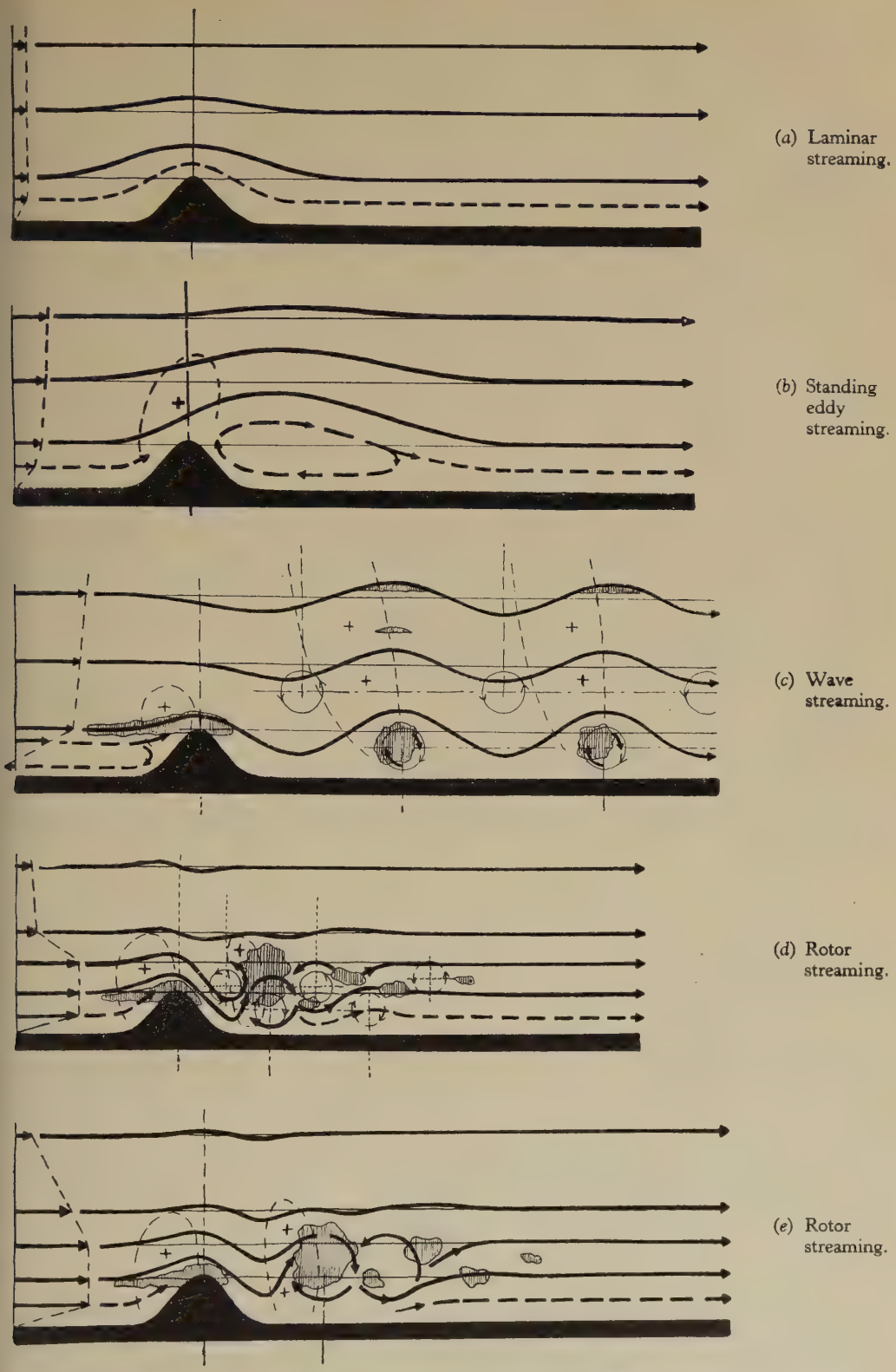


Figure 6. Types of airflow over ridges (after Förchtgott). Reproduced by courtesy of *Science Progress*.

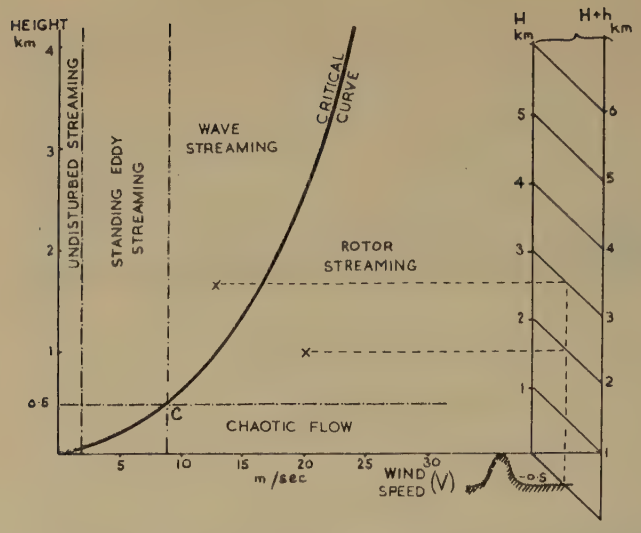


Figure 7. Diagram for determining type of mountain airflow (after Förchtgott).

but there is no provision for the length of the ridge. Something of this sort would be extremely valuable for aviation forecasting but for the time being we cannot regard Förchtgott's diagram as more than a convenient summary of his own ideas. Further intensive field work would be necessary before these results could be exploited with confidence. Förchtgott's diagram is reproduced as Fig. 7.

The critical curve was found empirically by Förchtgott from his own observations. The height (h) of a given mountain, say 0·5 km, is selected on the left-hand height scale. A line through this point parallel to the abscissa determines the critical point C. The ordinate through C is then drawn, so dividing the diagram into areas corresponding to the different flow types. For a given airstream the height (H) of the streaming layer above crest level, using the right-hand height scale, is then plotted against wind speed and the location of the resulting point in the diagram indicates the expected flow type.

e.g., $h = 0{\cdot}5$ km, $H = 2{\cdot}5$ km, $V = 13$ m/sec, produces wave streaming;
whilst, $h = 0{\cdot}5$ km, $H = 1{\cdot}5$ km, $V = 20$ m/sec, produces rotor streaming.

The remainder of the diagram at the right is merely a device for obtaining H in terms of the total depth of the streaming layer and the mountain height.

There is much in Förchtgott's results to confirm the predictions of perturbation theory as obtained by Scorer. The stable stratification and increase of wind with height implied for wave streaming (Fig. 6 (c)) means a decrease of the parameter l^2 with height, as required by perturbation theory. The wind profile for Förchtgott's rotor streaming (Figs. 6 (d) and 6 (e)) involves a large increase of l^2 with height. In such a case perturbation theory tells us that steady laminar streaming with lee waves is not possible but is unable to indicate what type of flow occurs in its place. Förchtgott, too, does not find lee waves but has suggested the alternative from his field data.

6. Aviation applications

Although the state of knowledge of mountain airflow is far from being complete and satisfactory, sufficient is now known to justify some cautious and tentative applications to aviation problems, including forecasting. Experience in other fields suggests that attempts at the practical application of new work can often assist by throwing fresh light

on aspects which are only partially understood, so making possible further advances. A few remarks are, therefore, given below on the possibilities, difficulties and limitations of applying current knowledge on mountain airflow to aviation problems.

(*a*) *Aviation forecasting*

The theoretical studies reviewed in Section 4 included streamlines calculated for a number of special cases. It should be said at the outset that similar calculations for real cases would be totally impracticable as a normal routine in a forecast office. Firstly it would be necessary to determine a model airstream which approximated to the real airstream and was at the same time suitable for the calculations. Secondly, the computation of the streamlines would be extremely laborious and lengthy, even for the case of the flow over a simple symmetrical ridge.

The most that could be done in practical forecasting would be to exploit, mainly qualitatively, some of the general results which emerge from the perturbation theory studies and in particular to make use of Scorer's deductions regarding the significance of the variations in the vertical of his parameter l^2. There has, of course, been no extended study of reported cases of standing waves aimed at testing the validity of these deductions, but in a few reported cases where there were sufficient details to permit such an examination (see for example, Scorer 1951), the l^2 profile has been calculated and the theory verified. Furthermore, if there are no large discontinuities of lapse rate, Scorer's criterion for waves reduces to a requirement for a strong forward wind shear in the air current across the ridge or mountain. This general property of airstreams containing waves is such a common feature in reported cases that there is in effect a good deal of circumstantial support for the theory. In addition it will be remembered that Förchtgott's field studies revealed the same feature.

The reports of wave effects contributed by B.E.A.C. pilots, a few of which were quoted in Section 2, also permit a rough check of the theory. In practically all these cases where it is possible to compute a representative l^2 profile, a pronounced maximum of l^2 is found through some lower layer, with very much smaller values above. The reduction is usually substantial, the value of l^2 in the upper layers being on average only about 1/9 of the maximum lower down. It is hoped to analyze and discuss these results more fully in a separate paper.

In view of the above considerations it now seems that an attempt could be made to indicate to pilots when waves might be expected to operate and give rise to large areas of lift or sink for flights crossing, or passing to the lee of mountain areas. When perturbation theory indicates that steady laminar, wavelike flow to the lee of mountains cannot occur, the forecaster can at present do little more than make guarded use of the results of Förchtgott. With regard to the computation of the profile of l^2, a convenient method, in which a celluloid scale is applied to the appropriate tephigram, has now been devised by Wallington (1953).

One great difficulty in bridging the gap between the theoretical results and practical forecasting arises from topographical reasons. The theory necessarily deals with simple mountain shapes, mostly infinite ridges; in practice something approaching the ideal obstacle occurs only at a few rare locations. In general the forecaster is concerned with what can only be described as irregular mountainous or hilly terrain. Over such areas on a day favourable for wave motions, the actual airflow must be the complex result of interference between many separate wave trains and for this reason alone the forecaster has little hope of saying much more than that areas of lift and sink will be encountered by a pilot. He cannot indicate the magnitude or horizontal extent of vertical currents on the basis of what is known at present.

(b) *Safety heights*

It is abundantly clear that, because of the vertical motion which may occur in air-streams over and to the lee of mountains, safety heights applicable to air routes crossing mountains should provide for a greater margin of safety than is normal over level terrain. The large number of unexplained or inadequately explained aircraft accidents in mountainous country adds point to this need. Tentative proposals as to what increases in safety heights should be applied have already been put forward, e.g. M.C.A.P. (1952) and Mason (1954), but these are very much in the nature of interim suggestions and have no very precise basis. A discussion of the question of aircraft safety heights is perhaps not appropriate to this *Journal* and, in any event, although it constitutes an important operational problem for aviation, it does not appear possible to devise a rigorous solution from existing knowledge. However, it may be mentioned that, apart from the characteristics of the airstream and of the terrain, the speed and course of the aircraft also have a bearing on the involuntary losses of height which may occur. The difficulty of proposing any precise determination of the necessary and sufficient safety height for any particular flight arises partly because our knowledge of mountain airflow does not enable us yet to make quantitative predictions of the vertical currents and their horizontal extent and partly because mountainous terrain in practice is complicated and irregular. The problem has been discussed further by Corby (1953). It is probable that some form of compromise solution will have to be accepted.

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The microwave properties of precipitation particles

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SUMMARY

The theory of scattering and attenuation by rain, snow and cloud is reviewed and theoretical results are presented in the form of equations, tables and graphs, so that the radar response to meteorological particles can be calculated at six wavelengths (10, 5·7, 3·2, 1·8, 1·24 and 0·9 cm) and various temperatures. Particular emphasis is placed on developments since Ryde's comprehensive paper in 1946. Published experimental results are compared with the theory.

All results computed from the theory are contained in Tables 4 and 5. The attenuation by water vapour and oxygen is given in an Appendix.

1. INTRODUCTION

In the comparatively new subject of radar-meteorology, the theoretical paper by Ryde (1946) stands out. In it he predicted the back-scatter and attenuation by meteorological phenomena, and, in almost every respect, the results obtained in experiments have confirmed the predictions. The present contribution will review the theoretical problem in the light of developments in the years that have elapsed since Ryde's work was done, and with reference to the experimental results.

Meteorological radar equipment provides information about the structure or pattern and about the quantity of precipitation. This paper will not be concerned with the structural information obtained, but will consider the scattering from a small region in which the properties of the precipitation can be taken to be uniform, and the attenuation taken along a path of propagation as a whole.

Plane electromagnetic waves travelling through air containing precipitation are scattered and absorbed by the particles of ice, snow or water. Water, with its larger dielectric constant, scatters more strongly than ice; in addition, it has a much larger dielectric loss and therefore the attenuation due to thermal dissipation is much greater for water particles than for ice particles.

Three quantities with the dimensions of area are derived for a particle in the path of a plane travelling wave. The 'back-scatter cross-section' σ is defined in such a way that σ multiplied by the incident intensity would be the total power radiated by an isotropic source which radiates the same power in the backward direction as the scatterer. The 'scattering cross-section' Q_s is such that Q_s, multiplied by the incident intensity, is the power scattered by the particle. The 'total absorption cross-section' Q_t is such that Q_t times the incident intensity is the total power taken from the incident wave.

The scattering properties of precipitation have made possible extensive use of microwave radar sets for meteorological purposes. The radar signal from precipitation is, however, not steady like that from a point target. The received power at any instant is made up of the resultant of the signals from a very large number of particles, and depends on their exact arrangement in space, which is continually changing. Marshall and Hitschfeld (1953) have shown that an instantaneous observation of received power gives very little information about the precipitation, and the signal at any given range has to be averaged until a large number of independent returns have been received. This can be arranged by using a long-persistence display or by photographic recording so that the returns from many transmitter pulses are combined.

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The equation for the average received power $\overline{P_r}$ is

$$\overline{P_r} = \frac{P_t A_e h}{8\pi r^2} \sum_1 \sigma \quad . \quad . \quad . \quad . \quad (1)$$

where P_t is the transmitted power,

h is the length of the pulse in space,

r the range from which the signal is being received,

A_e the equivalent antenna aperture,

$\sum_1 \sigma$ the total back-scatter cross-section of the particles in unit volume.

The received power is proportional to h so that the sensitivity of the system can be increased by lengthening the transmitter pulse. (If this change is accompanied by a reduction in receiver bandwidth, a further improvement in signal-to-noise ratio is obtained.) However, the range resolution is correspondingly reduced. Increasing A_e, on the other hand, improves both sensitivity and angular resolution, so that the antenna is made as large as practicable.

The equivalent aperture A_e is less than the actual area of the antenna for two reasons. Firstly, not all the power coming from the primary feed falls inside the parabolic reflector, neither is the distribution of power uniform across the aperture (see, e.g., Silver (1949) § 6.4). In normal radar systems the effective aperture A' for a point target is about 65 per cent of the actual aperture. The second reason, which applies only to distributed targets, affects weather radars. Some of the radiated power goes into side-lobes and the edges of the main beam, and though all of it strikes precipitation, some of the returned power comes from directions to which the antenna is less sensitive for reception. This means that the average receiving aperture A_e for all parts of the target is less than A', the value for the central portion. An exact value for A_e is difficult to determine, but a conservative estimate is probably 0·5 × (actual area).

The fact that the target normally fills the beam at any range accounts for the inverse square law for received power versus range, in place of the usual fourth-power law for a point target such as an aircraft. Though Eq. (1) does not mention wavelength explicitly, $\sum_1 \sigma$ increases rapidly as the wavelength decreases, as discussed in Section 3.

It will be shown in Section 3 (b) that for a wavelength λ long compared with the diameter D of the particles, the approximation

$$\sigma = \frac{\pi^5 D^6}{\lambda^4} |K|^2 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

can be used, where $|K|^2$ depends on the dielectric constant of the material and varies only slowly with wavelength (Section 2). From Eqs. (1) and (2) we have

$$\overline{P_r} = \frac{\pi^4 P_t A_e h}{8r^2 \lambda^4} \sum_1 D^6 |K|^2 \quad . \quad . \quad . \quad (3)$$

So far we have neglected the attenuating effect of any precipitation through which the signal may have to pass. The intensity of plane parallel radiation is reduced, in traversing distance dx through precipitation, from I_0 to I, where

$$I = I_0 \exp\left\{-\sum_1 Q_t \, dx\right\} \quad . \quad . \quad . \quad . \quad (4)$$

The summation is over all the particles in unit volume. In practical units, the attenuation per kilometre of path length is

$$\text{attenuation (db km}^{-1}) = 0{\cdot}4343 \sum_1 Q_t, \quad . \quad . \quad . \quad (5)$$

where the summation is over one cubic metre and Q_t is in cm^2. For the case of wavelength long compared to the drop diameters (Section 3 (b)), we can use an approximation for Q_t, Eq. (14), so that Eq. (5) becomes

$$\text{attenuation (db km}^{-1}) = 0{\cdot}4343 \frac{\pi^2}{\lambda} \sum_1 D^3 \, Im\,(-K)^* \quad . \quad . \quad (6)$$

$$= 0{\cdot}4343 \frac{6\pi}{\lambda} \frac{M}{\rho} Im\,(-K),$$

where M is the water content in gm m^{-3}, and ρ is the density of water in gm cm^{-3}. For radar, the radiation travels there and back, so the numerical constant should be doubled to obtain decibels per kilometre of range.

The complete equation for the average received power $\overline{P_r'}$ allowing for two-way attenuation is then

$$\overline{P_r'} = 10^{-0{\cdot}0869 \int_0^r \left(\sum_1 Q_t\right) dx} \frac{P_t A_e h}{8\pi r^2} \sum_1 \sigma \times 10^{-10} \quad . \quad . \quad (7)$$

where $\sum_1 Q_t$ and $\sum_1 \sigma$ are in cm^2 m^{-3}, A_e is in m^2, h is in m, and r is in km. The effect of atmospheric attenuation, which is not included in this expression, will be discussed in Appendix I.

2. Dielectric properties of water and ice

To calculate the scattering and attenuation of electromagnetic waves by particles in the atmosphere, one must know the values of the complex dielectric constant ϵ for the temperature and wavelengths under consideration.

The square root of ϵ is the complex refractive index m which may be written

$$m = n - j\kappa,$$

where n is the refractive index and κ the absorption coefficient of the material. In general, the scattering and attenuation are complicated functions of m (Section 3), but in the special case of wavelength long compared to diameter,

$$\text{back-scatter} \propto |K|^2 \quad . \quad . \quad . \quad \text{from (3)}$$

$$\text{attenuation} \propto Im\,(-K) \quad . \quad . \quad . \quad \text{from (6)}$$

where $K = \dfrac{m^2 - 1}{m^2 + 2}$.

For a given substance m is a function of wavelength and temperature. Saxton and Lane (1946) measured values of n and κ for water at a few wavelengths and calculated other values in the centimetric band, for temperatures from 0° to 40°C. Since then Lane and Saxton (1952) have reported additional measurements at a lower wavelength (6·2 mm) and some more accurately determined values at 1·24 and 3·2 cm. Also, they have determined values of κ for supercooled water down to − 8°C at three wavelengths. Measurements by Collie, Hasted and Ritson (1948) at 10, 3 and 1·25 cm, and Buchanan (1952) at 3·2 and 1·27 cm agree to within a few per cent with the results of Lane and Saxton.

* *Im* is used for ' imaginary part of.'

In Table 1, the variation of the refractive index of water with temperature and wavelength is shown, and the quantities $Im\,(-K)$ and $|K|^2$ have been calculated. Values for n and κ are taken from Kerr (1951) for 10 cm wavelength, and directly from Lane and Saxton (1952) for the other wavelengths.

Measurements of the quantities n and κ for ice show that in the centimetre band both n and κ are practically independent of λ. Dunsmuir and Lamb (1945) and Lamb (1946) found κ to be a very small quantity, ranging from 12×10^{-4} at 0°C, through $1{\cdot}9 \times 10^{-4}$ at − 20°C to $1{\cdot}3 \times 10^{-4}$ at − 40°C. More recent measurements by Cumming (1952) give values for κ roughly twice those of Lamb; Cumming's value at − 12°C agrees with one obtained at that temperature by the M.I.T. Dielectric Laboratory (Cumming 1952). Both Cumming, and Lamb and Turney (1949) agree on a value of $n = 1{\cdot}78$ for ice, constant over the temperature range 0 to − 20°C. The refractive index data for ice are summarized in Table 2.

TABLE 1. VARIATION OF REFRACTIVE INDEX OF WATER WITH TEMPERATURE AND WAVELENGTH

		λ (cm)			
		10	3·21	1·24	0·62
T (°C)					
20		8·88	8·14	6·15	4·44
10	n	9·02	7·80	5·45	3·94
0		8·99	7·14	4·75	3·45
− 8			6·48	4·15	3·10
20		0·63	2·00	2·86	2·59
10		0·90	2·44	2·90	2·37
0	κ	1·47	2·89	2·77	2·04
− 8				2·55	1·77
20		0·928	0·9275	0·9193	0·8926
10		0·9313	0·9282	0·9152	0·8726
0	$\|K\|^2$	0·9340	0·9300	0·9055	0·8312
− 8				0·8902	0·7921
20		0·00474	0·01883	0·0471	0·0915
10		0·00688	0·0247	0·0615	0·1142
0	$Im\,(-K)$	0·01102	0·0335	0·0807	0·1441
− 8				0·1036	0·1713

TABLE 2. VARIATION OF REFRACTIVE INDEX* OF ICE WITH TEMPERATURE (BASED ON DATA FROM CUMMING (1952))

T (°C)			
	n	1·78	At all temperatures
0		24×10^{-4}	
− 10	κ	$7{\cdot}9 \times 10^{-4}$	
− 20		$5{\cdot}5 \times 10^{-4}$	
	$\|K\|^2$	0·197	At all temperatures. This is for ice of unit density, the value to be used when D is diameter of melted ice particle, in Eq. (3). (Marshall and Gunn (1952)).
0		$9{\cdot}6 \times 10^{-4}$	
− 10	$Im\,(-K)$	$3{\cdot}2 \times 10^{-4}$	
− 20		$2{\cdot}2 \times 10^{-4}$	

* Refractive index of ice is independent of wavelength in the centimetre band.

Snow may be considered as a mixture of two dielectrics, ice and air. According to Debye (1929), one can calculate the ratio K/ρ for a mixture of two dielectrics by adding the K/ρ values for the two substances in proportion to the mass of each. Neglecting the contribution of K/ρ for air, then K/ρ for ice-air mixtures of any density is practically constant. Cumming (1952) made measurements of the dielectric constant of snow of various densities. Using his data one finds that K/ρ varies more or less uniformly from 0·46 for snow of density 0·917 g cm^{-3} to 0·50 for density 0·22 g cm^{-3}.

3. Spherical particles

(a) *Mie's theory*

A complete theory for spherical particles of any material in a non-absorbing medium was developed by Gustav Mie (1908). Mie's work has been restated by Stratton (1941), Goldstein (1946) and by Kerr (1951). Using the notation of these later workers, the cross-sections of a spherical particle are

$$\sigma = \frac{\lambda^2}{4\pi} \left| \sum_{n=1}^{\infty} (-1)^n (2n+1)(a_n - b_n) \right|^2 \quad . \quad . \quad (8)$$

$$Q_s = \frac{\lambda^2}{2\pi} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2) \quad . \quad . \quad . \quad (9)$$

and

$$Q_t = \frac{\lambda^2}{2\pi} (-Re) \sum_{n=1}^{\infty} (2n+1)(a_n + b_n)^* \quad . \quad . \quad . \quad (10)$$

where λ = wavelength in the medium,

a_n = coefficient of the nth magnetic mode,

b_n = coefficient of the nth electric mode.

The coefficients a_n and b_n are made up of Spherical Bessel functions of order n. The arguments of the functions are α and m and contain the properties of the sphere and the wavelength; $\alpha \equiv \frac{\pi D}{\lambda}$, where D is the diameter of the sphere, and m is the complex refractive index of the material discussed in Section 2.

Eqs. (9) and (10) have been computed by Lowan (1949) for water spheres at 18°C for various wavelengths between 3 mm and 10 cm.

(b) *Small spheres (Rayleigh approximation)*

By expanding the a's and b's in ascending order of α for $n = 1, 2 \ldots$, formulae for Q_s and Q_t can be obtained which apply to small values of α. Neglecting terms of higher than sixth power of α, only a_1, b_1 and b_2 are significant, and they are

$$a_1 = -\frac{j}{45}(m^2 - 1)\alpha^5 \quad . \quad . \quad . \quad . \quad . \quad . \quad (11a)$$

$$b_1 = -\frac{2j}{3}\frac{m^2-1}{m^2+2}\alpha^3 \left(1 + \frac{3}{5}\frac{m^2-2}{m^2+2}\alpha^2 - \frac{2j}{3}\frac{m^2-1}{m^2+2}\alpha^3\right) \quad . \quad (11b)$$

$$b_2 = \frac{j}{15}\frac{m^2-1}{2m^2+3}\alpha^5. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11c)$$

* *Re* signifies 'real part of.'

For diameters small compared with the wavelength, we can obtain approximate formulae for σ and Q_s by putting $\alpha \ll 1$ into these equations. Then, only the leading term of b_1, the coefficient of the first electric mode, need be considered, and the contribution to σ and Q_s of all higher order terms in the a's and b's can be neglected. The resulting formulae are known as the Rayleigh approximations :

$$\sigma = \frac{\lambda^2}{2\pi} \cdot 2\alpha^6 \left|\frac{m^2-1}{m^2+2}\right|^2 = \frac{\pi^5 |K|^2}{\lambda^4} D^6 \quad . \quad . \quad . \quad (12)$$

$$Q_s = \frac{\lambda^2}{2\pi} \cdot \frac{4}{3} \alpha^6 \left|\frac{m^2-1}{m^2+2}\right|^2 . \quad . \quad . \quad . \quad . \quad (13)$$

The total absorption cross-section Q_t is given by Eq. (10) in which the real parts of the coefficients a and b have to be obtained. In this case, however, it does not always happen that the leading term in b_1 contributes the most to Q_t. Indeed, if m is purely real, the α^3 and α^5 terms of b_1 are imaginary and contribute nothing to Q_t, and of the terms listed in Eqs. (11) only the α^6 term of b_1 will appear in Q_t.

At this stage it is convenient to introduce a fourth cross-section Q_a, the absorption cross-section, to account for the power taken from the incident wave which is dissipated internally as heat and not scattered; it is simply $(Q_t - Q_s)$. Putting $\alpha \ll 1$, the contribution to Q_t of the α^6 term in b_1 is the same as the leading term in Q_s, so that this term does not appear in Q_a. Then the leading term in Q_s is given by Eq. (13) and the leading term in Q_a comes from the α^3 term in b_1 and is given by

$$Q_a = \frac{\lambda^2}{2\pi} \cdot 2\alpha^3 \, Im\left(-\frac{m^2-1}{m^2+2}\right) \text{ for } \alpha \ll 1 \quad . \quad . \quad . \quad (14)$$

and, of course,

$$Q_t = Q_a + Q_s. \quad . \quad . \quad . \quad . \quad (15)$$

For a pure ' lossless ' dielectric $Q_a = 0$ so that $Q_t = Q_s$ and is given by Eq. (13). For a very ' lossy ' dielectric like *water* and particle sizes such that $\alpha \ll 1$, $Q_s \ll Q_a$ so that $Q_t \simeq Q_a$ and is given by Eq. (14). For *ice* at 0°C and in the size range of snowflakes, at centimetre wavelengths, Q_a and Q_s are comparable and both must be calculated; Q_t is their sum.

In the various published treatments of the problem of spherical particles, a few errors have been noticed. These are discussed in Appendix II.

(c) *Larger spheres*

F. T. Haddock (1948) has computed for water at 18°C the ratio between Q_t, calculated from Lowan's (1949) Bureau of Standards tables, and Q_t, calculated from Eq. (14), assuming $Q_t = Q_a$. This ratio Q_t (Mie)/Q_t (Rayleigh) is plotted in Fig. 1 for various wavelengths. Ryde's (1946) Fig. 3 is a similar plot, but of

$$f_a = \frac{Q_t \text{ (Mie)}}{\text{geometrical cross-section}}$$

for a wavelength a little over 1 cm.

The region of α in which the exact value of Q_t given by Eq. (10) and the approximate value given by Eq. (15) [i.e. by (13) and (14)] are nearly equal we have called the ' Rayleigh region for attenuation.' The diameters of cloud particles are well inside this region for all wavelengths down to 3 mm. Rain will be discussed in Section 4.

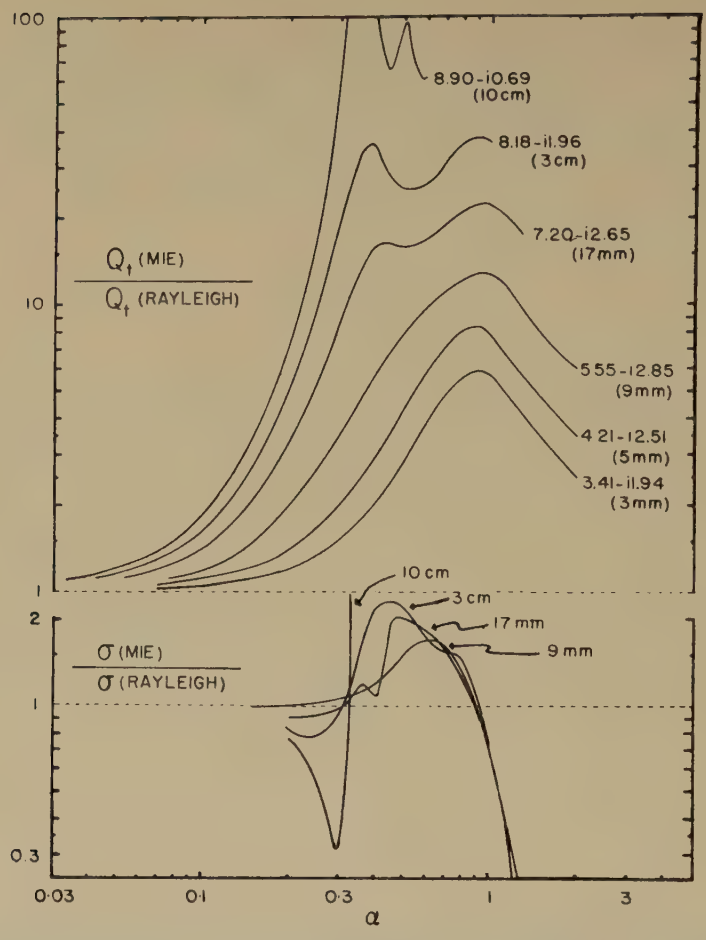

Figure 1. Ratio of actual attenuation (upper) and actual back-scattering (lower) to that given by the Rayleigh approximation, for water at 18°C. In the upper diagram the complex refractive index and wavelength (in parenthesis) are shown for each curve.

The corresponding situation for back-scatter is shown also in Fig. 1. Here σ (Mie)/σ (Rayleigh) is plotted against α for various wavelengths. We shall define the ' Rayleigh region for back-scatter ' as the region for which this ratio lies between $\frac{1}{2}$ and 2. It will be seen that for wavelengths up to 17 mm, the ratio stays within these limits up to about $\alpha = 1$, where it falls rapidly. However, at longer wavelengths the ratio first rises beyond 2 so that the region only extends to $\alpha = 0{\cdot}4$ or less.

At 17 mm wavelength, the Rayleigh region as defined here extends to 6 mm diameter and includes all normal raindrop sizes. At shorter wavelengths, the limit occurs at smaller diameters, so that at $\lambda = 3$ mm, only particles less than 1 mm diameter are inside it. At 3 cm wavelength, the region is also reduced, extending only to 3 mm diameter, but at $\lambda = 10$ cm it covers all raindrop sizes so that the approximate formula can be safely used on rain.

Data for ice, equivalent to those plotted in Fig. 1 for water, were not evaluated by Lowan. Ryde (1946), however, considers that the Rayleigh approximate expressions for ice may be used for values of α up to 0·5 with small error. This means that the Rayleigh expressions (12), (13) and (14) may be used to compute the back-scattering cross-section

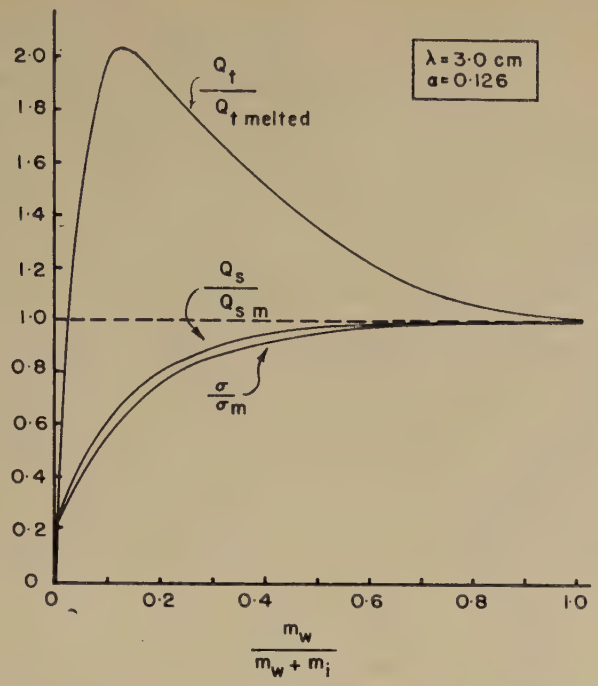

Figure 2. Attenuation, scattering and back-scatter cross-sections of water-coated ice sphere vs. proportion of water by mass. The cross-sections are relative to those for a water sphere of equal total mass.

and the attenuation for ice cloud at all centimetre wavelengths, and for single snow crystals probably down to wavelengths of 1·25 cm. Considering aggregate snowflakes as spherical (non-sphericity is found to be not too important, see Section 5 (a)), the Rayleigh approximate expressions can be applied at wavelengths down to about 5 cm for all except unusually large flakes. Scattering and attenuation from snow are discussed in Section 5.

(*d*) *Water-coated ice spheres*

Aden and Kerker (1951) extended Mie's theory to concentric spheres. Langleben and Gunn (1952) have applied this extended theory to spheres of ice coated with water in the region $\alpha < 1$. They consider a sphere, originally all ice, which changes gradually to ice coated with water, and finally entirely to water, maintaining constant mass. The calculated cross-sections for such a particle are plotted against the ratio of the mass of water to total mass in Fig. 2 for 3 cm wavelength and 1·2 mm initial diameter. As the thickness of the water coating increases from zero, the quantities σ, Q_s and Q_t all increase rapidly from their values for ice; when the sphere is still 70 per cent ice (so that the water thickness is less than one-tenth of the total radius) σ and Q_s are already nearly equal to those for the water sphere. Earlier still, Q_t reaches a maximum value, about twice as great as that for water. No simple explanation has been offered as yet for this extremely rapid increase of the attenuation to twice the value for pure water. For other wavelengths and diameters in the Rayleigh region, the curves for Q_s, σ and Q_t are very similar to those of Fig. 2. For larger values of α, the scattering cross-sections do not rise so early, and Q_t does not show such an extreme behaviour.

4. Rain

(*a*) *Drop-size distributions*

Where Rayleigh theory is applicable, radar can be used to measure the quantity $Z = \sum_1 D^6$, the sum of the sixth powers of the drop diameters in unit volume. A more significant quantity meteorologically is the rainfall rate, $R = \frac{\pi}{6} \rho \sum_1 (v_D D^3)$, where v_D is the terminal velocity of a drop of diameter D. To relate the two quantities the drop-size distribution must be observed. Distributions for rain at the ground have been determined in various parts of the world, for example, by Laws and Parsons (1943), Wexler (1948), Marshall and Palmer (1948), Best (1950), Hood (1950). Most of these workers have calculated simultaneous values of Z and R from the distribution of drop sizes obtained on a horizontal surface. Then from a number of such simultaneous values plotted on log-log paper a locus of best fit for the data of the form $Z = aR^b$ has been computed. There have been many such Z-R relations quoted in the literature in the past ten years, which give a wide range of Z values for a given R. Out of these, the authors have found ten relations which are based on measurements using original drop-size distribution data. These, along with pertinent information, are included in Table 3.

TABLE 3. Relations between Z and rainfall rate

$Z = aR^b$	Author	Location of observations	Ranges of rainfall rates treated	Number of Z-R values	How locus determined
$290\ R^{1.41}$	Blanchard (1953)	Hawaii	0·28 to 127 mm hr^{-1}	28	least squares fit
$208\ R^{1.58}$	Wexler (1948) on Anderson *et al.* (1947) data	Hawaii	7 to 50 mm hr^{-1}	49	least squares fit
$269\ R^{1.48}$	Mt. Washington Observatory (1951)	Cambridge, Mass.	0·07 to 87·5 mm hr^{-1}	63	least squares fit
$214\ R^{1.58}$	Wexler (1948) on Laws and Parsons' (1943) original data	Washington, D.C.	0·37 to 114 mm hr^{-1}	98	least squares fit
$200\ R^{1.6}$	Revised from Marshall and Palmer (1948)	Ottawa	0·15 to 35 mm hr^{-1}	250	best fit by eye (with a check to centre locus amongst data at that slope)
$295\ R^{1.21}$	Hood (1950)	Ottawa	0·5 to 60 mm hr^{-1}	270	best fit by eye
$505\ R^{1.44}$	Best (1950)	Shoeburyness	0·18 to 4·2 mm hr^{-1}	90	from analysis of mean distributions
$257\ R^{1.48}$	Best (1950)	Ynyslas	0·36 to 8.9 mm hr^{-1}	299	
$436\ R^{1.46}$	Best (1950)	East Hill	0·41 to 25·1 mm hr^{-1}	149	
$127\ R^{1.29}$	Twomey (1953)	Sydney, Australia	0·2 to 9 mm hr^{-1}	34	least squares fit

The number of points used to make up the different loci varies from one worker to another as does the range of rainfall rates covered. The great majority of the points in each case lie between 1 and 10 mm hr^{-1}.

The first six relations, obtained in North America lie within a factor 1·55 of the locus $Z = 244\ R^{1.55}$. The next three, obtained from measurements made in Great Britain, lie within a factor 1·59 of the locus $Z = 376\ R^{1.50}$. These nine relations are included within a band, a factor 1·70 either way from the locus

$$Z = 353\ R^{1.52} \quad . \quad . \quad . \quad . \quad (16)$$

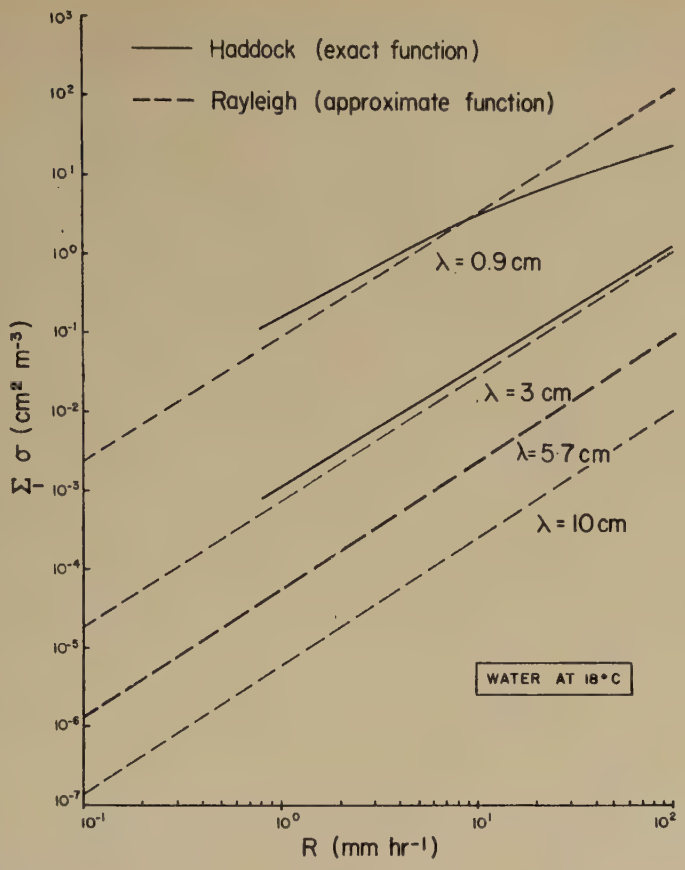

Figure 3. Exact and approximate (Rayleigh) back-scattering cross-sections per unit volume of rain-filled space, plotted against rainfall intensity.

The tenth relation $Z = 127\,R^{2\cdot29}$, reported by Twomey (1953) from measurements in Australia, departs considerably from the nine others, with a much steeper slope. His locus is based on relatively few data and while it does pass through the region defined by Eq. (16) in the important rainfall rate range 1-10 mm hr^{-1}, its departure at other rainfalls indicates the difficulty of giving a universal Z-R relation for all locations.

Any one of these loci represents only the average relationship at that particular place. Quite apart from scatter due to sampling methods which could easily amount to a factor 1·5 either way, the size distribution for a given R undoubtedly varies from one sample to another. Thus if one uses an average relationship such as Eq. (16), it is probably good to no better than a factor 2 or 3 either way. Unless one takes the variation during a rainfall and from one location to another into account, this uncertainty has to be accepted.

(*b*) *Scattering by rain*

The curves in Fig. 1 showed the behaviour of the exact back-scattering cross-sections for single drops. Haddock, in Plate 6 of his unpublished report, has evaluated the exact cross-sections for rain of various intensities using the drop-size distribution results of Laws and Parsons (1943). Haddock's data for back-scattering have been replotted in Fig. 3 which shows $\Sigma_1 \sigma$ against rainfall rate for two wavelengths, 3 cm and 0·9 cm. For

comparison, the Rayleigh approximate back-scattering cross-sections are shown for these same wavelengths. They are derived from summing Eq. (12) over unit volume and substituting $\sum_1 D^6 = 200\,R^{1.6}$.

Haddock does not give a plot of the exact $\sum_1 \sigma$ for wavelengths greater than 3 cm, but the 5·7 and 10 cm curves will be identical with the Rayleigh one at all but the very heaviest rainfalls. At 0·9 cm, the exact back-scattering cross-section is larger than the Rayleigh for low rainfalls; it is the same at a rainfall of about 10 mm hr^{-1} and then becomes less quite quickly at higher rainfalls, being some 7·5 db below the Rayleigh value at 100 mm hr^{-1}. This rapid divergence of the two functions at 0·9 cm for heavy rainfalls is due to the rapid descent of the σ-curve for that wavelength (Fig. 1), at values of α which correspond to raindrop diameters. However, wavelengths shorter than 3 cm are not very useful for rain observations because of attenuation. At 3 cm wavelength, the appropriate values of α are somewhat lower, so that the difference between the exact curve and the Rayleigh curve does not exceed 2 db.

The curves of Fig. 3 are for a temperature of 18°C. In the Rayleigh region, temperature change will not affect the scattering greatly, since the quantity $|K|^2$ to which the scattering is proportional does not vary by more than a few per cent. Out of the Rayleigh region, the scattering is a more complex function of the refractive index, but according to Ryde (1946) the temperature effect on scattering is still not more than a few per cent.

Several workers have performed experiments to confirm Eq. (3). Marshall, Langille and Palmer (1947) obtained proportionality between $\overline{P_r}$ and $\sum_1 D^6$ at 10 cm wavelength, but did not measure the absolute power level. Hooper and Kippax (1950) confirmed the wavelength dependence at 10 cm, 3 cm and 1·25 cm, and the proportionality to pulse length h. They also reported that the absolute value of received power averaged about 1 db above the theoretical value. Hood (1950) found the expected proportionality between $\overline{P_r}$ and $\sum_1 D^6$, but $\overline{P_r}$ was 3 or 4 db too low at 10 cm and 7 db too low at 3 cm. He attributed this to a fixed error in the radar calibration or in the method of measuring $\sum_1 D^6$. However, Austin and Williams (1951) calibrated a radar with a known spherical target, thus eliminating the need for power measurements, and still obtained results 4 to 10 db too low on rain at 10 cm. They also ascertained that Hooper and Kippax were measuring *peak* received powers instead of the *average* power $\overline{P_r}$, which must have been several decibels lower. Donaldson *et al.* (1953) working with a 1·25 cm radar, which was calibrated with a spherical target, have reported a 2·5 db discrepancy between $\overline{P_r}$ and $\sum_1 D^6$. Each of these experiments shows considerable scatter of data, so the experiments cannot be said to disagree significantly with each other. However, it does seem that they disagree with the theory, and no satisfactory explanation has been reached.

(c) *Attenuation by rain*

For rain, the Rayleigh approximation for attenuation only applies for wavelengths of 10 cm and longer. At the shorter wavelengths, the attenuation through rain is no longer given by Eq. (6), but depends explicitly on diameter as well as on water content.

The data published by Laws and Parsons have been used by Ryde (1945, 1946) and Haddock to compute the attenuation at various wavelengths by a given rate of rainfall. The authors have repeated some of Haddock's calculations, using Laws and Parsons' data as plotted by Rigby, Marshall and Hitschfeld (1954) and Haddock's ratio in Fig. 1.

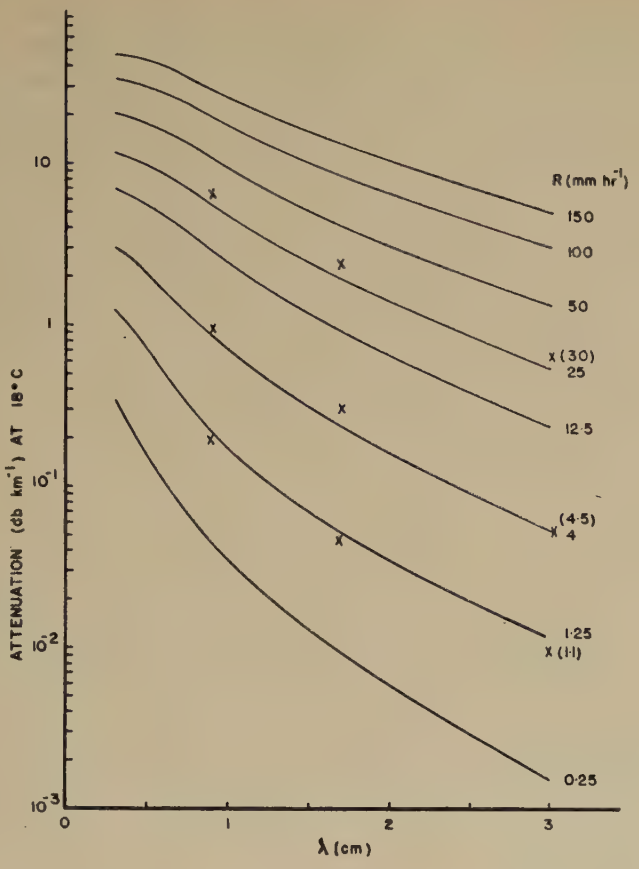

Figure 4. Attenuation versus wavelength for rainfall of various rates. Solid curves plotted from data in Haddock (1948); crosses are values computed by the authors.

In Fig. 4 these various calculated attenuations are plotted against wavelength for several rates of rainfall. Ryde's (1945) report shows good agreement with Haddock at rainfall rates above 12·5 mm hr^{-1} but his figures are up to 30 per cent greater at the lower rainfalls.

The results of Fig. 4 are replotted in Fig. 5. Here the abscissa is rainfall rate in mm/hr and the ordinate is attenuation divided by rainfall rate. The curve for 10 cm is plotted from Ryde's (1945) report; he gives no figures for wavelengths between 3·2 cm and 10 cm so the curve for 5·7 cm was obtained by graphical interpolation of his results. This process may have introduced an error of about 20 per cent. The dashed lines are the result of applying the Rayleigh approximation (Eq. 6). The ordinate in this diagram is the factor k in Ryde's (1946) Section 7; he suggests that, for many purposes, it may be taken as constant for a given wavelength. At 10 cm, k is very nearly constant, but has such a small value that it is negligible for radar purposes. At 5·7 cm, k increases slowly with rainfall rate; $k = 0{\cdot}003$ is an average figure, the empirical equation $k = 0{\cdot}0022\,R^{0{\cdot}17}$ fits more closely. At 3 cm, k is by no means constant and can be represented reasonably well by the empirical relation $k = 0{\cdot}009\,R^{0{\cdot}3}$, where R is the rainfall rate in mm hr^{-1}. However, if the rainfall rate is known to lie in a restricted range on any particular occasion, a suitable mean value for k could be used. At 9 mm, k is very nearly constant again, although it is 5 to 10 times the corresponding Rayleigh value. This is because the ratio plotted in Fig. 1 is near its maximum for these drop

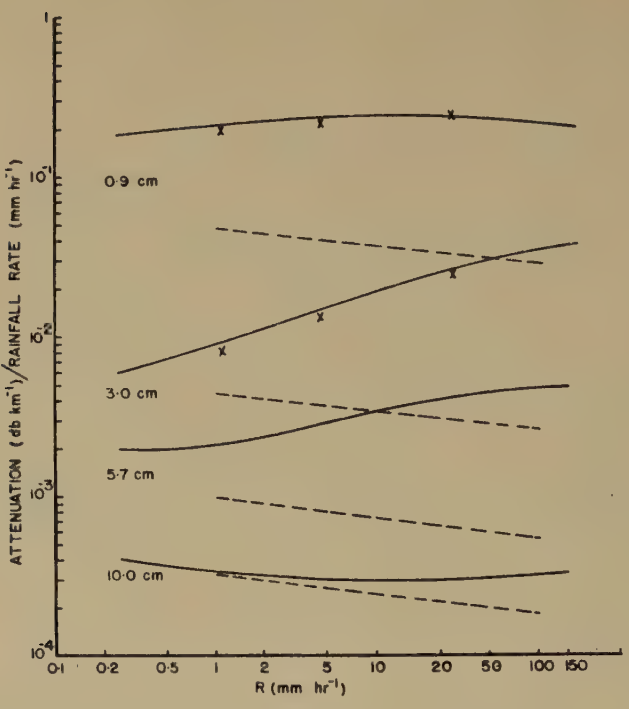

Figure 5. Attenuation/rainfall rate versus rainfall rate for various wavelengths. The solid curves are taken from Haddock (1948) for 0·9 and 3·0 cm, from Ryde (1945) for 10 cm. The curve at 5·7 cm is interpolated from Ryde's data. Dashed lines show the Rayleigh approximation.

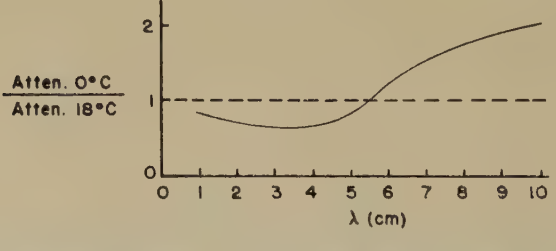

Figure 6. Approximate temperature correction curve for attenuation by rain (Fig. 7 of Ryde (1946)). For exact corrections at various rainfall rates, see Ryde (1945).

sizes at this wavelength. Whenever k can be taken as constant, the attenuation in dB over the path is simply k times the integral of rainfall rate along the path in km-mm hr^{-1}.

Since the refractive index of water varies with temperature, the attenuation does also. In the Rayleigh region, attenuation is proportional to $Im(-K)$ and roughly doubles in value in going from 18°C to 0°C. Outside the Rayleigh region, to determine the attenuation at a temperature other than 18°C, it is necessary to have a set of curves of Q_t (Mie)/Q_t (Rayleigh), similar to those of Fig. 1 for the refractive indices of water at the required wavelengths at the new temperature. This involves lengthy computations which Ryde (1945) carried out for a number of temperatures. Surprisingly, the attenuation at wavelengths less than 5·5 cm decreases in going from 18°C to 0°C. Ryde used the refractive index values quoted by Saxton and Lane (1946). The present authors made approximate calculations using the more recent values of Lane and Saxton (1952) which

also gave a decrease at these wavelengths; but only of about 10 per cent as opposed to Ryde's 30 per cent. Ryde's (1946) approximate correction curve which applies to rainfall rates greater than 2·5 mm hr^{-1} is shown in Fig. 6.

A number of point-to-point attenuation measurements have been made. Robertson and King (1946) measured attenuation at 1·09 and 3·2 cm wavelength in rainfalls from 2 to 100 mm hr^{-1}; their results show good agreement with Haddock's data, except below about 10 mm hr^{-1}, where measurements become inaccurate. Mueller (1946), at 0·62 cm, made similar measurements which are everywhere within 10 per cent of the theoretical values. The attenuations measured by Anderson, Day, Freres and Stokes (1947) in Hawaiian orographic rain at 1·25 cm are consistently greater than predicted values by factors of from 1·5 to 2. This may be due to drop-size distributions which differ markedly from those of Laws and Parsons. This seems a possible explanation in view of the remarkably large median diameters quoted by Anderson *et al.*, though Wexler (1948) found that their data gave a Z-R relationship similar to that computed from Laws and Parsons data (Table 3).

(*d*) *Distorted raindrops*

Photographs of water drops falling through stagnant air taken by Magono (1954) in the laboratory show that the bottom of a falling drop is always flattened (for drops of equivalent spherical diameter 2 mm and more) and the larger the drops the flatter the shape. Similar flattening has been observed in the few photographs so far taken of actual falling raindrops (Jones and Dean 1953). Experiments by Blanchard (1948) have shown that when a large drop catches up and collides with a smaller one it will oscillate about an equilibrium shape. Such oscillations can also occur as a result of aerodynamic forces on the drop as it falls. At any time, therefore, a fraction of the total number of drops in the contributing region is non-spherical. Stevenson (1953a, b) has recently published a theory of scattering by ellipsoids analogous to Mie's theory for spheres, and finds that for spheroids the scattered field can be expressed as a convergent series in α involving only elementary functions of the refractive index and of the geometry of the problem. This work has not yet been applied to calculations of Q_t and σ for non-spherical particles. An approximate theory for small spheroids which is analogous to the Rayleigh approximation has been used by Atlas, Kerker and Hitschfeld (1953) however, and they find that the flattening diminishes scattering and attenuation with vertical polarization and enhances it with horizontal polarization.

This effect cannot account for the difference between measured and predicted average received power because the observations were made with horizontally polarized radars, which would lead to greater measured powers rather than smaller.

5. Snow

(*a*) *Scattering by snow*

Dry snow consists of ice crystals, either single or aggregated. Calculations by Atlas, Kerker and Hitschfeld (1953) and by Labrum (1952b) on spheroids indicate that even considerable departures from sphericity are unimportant in the case of ice particles, the cross-sections never being more than twice as great as those for spheres of the same mass. Since the refractive index for ice is less than that of water, the factor $|K|^2$ to which scattering in the Rayleigh region is proportional, is about 0·22 times that for a water sphere of the same mass.

Aggregate snowflakes consist of a mixture of air and ice of low mean density. It can be shown (Marshall and Gunn 1952) that K/ρ is the same as that for the same mass

of ice in compact form. This means that, for snowflakes small enough for the Rayleigh approximation to apply, $\sum_1 \sigma$ can be calculated by catching a sample of snow flakes, melting them, measuring $\sum_1 D^6$ for the resulting drops and allowing for the dielectric factor.

This was done by Langille and Thain (1951) who also made radar measurements at the same time. The agreement with the form of Eq. (1) was within the scatter of the observations, but the received power was 4 db less than predicted. As in the rain measurements, this difference is unexplained. Langille and Thain's results, on further analysis (Marshall and Gunn 1952), yield an empirical relation between Z and R, where Z refers to unit volume of snowstorm and is the sum of the sixth powers of the diameters of the drops formed from the melted flakes, and R is the snowfall rate at the ground measured in millimetres of water per hour. The overall relation is $Z = 200\,R^{1\cdot6}$, identical with the Canadian results for rain*. Since the reflectivity of ice is 0·22 times that of water, radar echoes from snow are 0·22 times those from rain of the same rainfall rate.

(b) *Attenuation by snow*

Attenuation by snow will be partly due to scattering and partly due to absorption of the incident energy (Section 3 (b)).

According to Eqs. (5) and (15)

$$\text{attenuation (db km}^{-1}) = 0\cdot4343 \sum_1 Q_t$$
$$= 0\cdot4343 \left(\sum_1 Q_s + \sum_1 Q_a \right).$$

We can use the approximate expressions (13) and (14) if the wavelength is long compared to the particle diameters; the values of $|K|^2$ and $Im\,(-K)$ which apply are listed in Table 2 and are based on Cumming's (1952) values. This gives

$$\text{attenuation at 0°C (db km}^{-1}) = 0\cdot00349 \frac{R^{1\cdot6}}{\lambda^4} + 0\cdot00224 \frac{R}{\lambda}. \qquad (17)$$

Ryde (1946) suggests that attenuation by snow in the Rayleigh region and at centimetric wavelengths is given by the second term only. In Table 5, values of the attenuation calculated from Eq. (17) for 3 temperatures show that the first term (scattering) is indeed negligible at 10 cm but becomes increasingly important on going to shorter wavelengths.

(c) *Wet snow and the 'bright band'*

Atlas, Kerker and Hitschfeld (1953) show that randomly-oriented small water spheroids scatter and attenuate more strongly than water spheres of the same volume; for example, the increase is by a factor of 20 or more if the length of each spheroid is 10 times its diameter. Labrum (1952b) has computed the scattering by an ice ellipsoid coated with water for wavelengths large compared with the size of the particle (corresponding to the Rayleigh approximation for spheres). He showed that even for comparatively thin coatings the composite particle should scatter nearly as well as an all-water particle of the same shape. In a subsequent experiment, he confirmed this result (Labrum 1952a).

Extrapolating from the results of Labrum for small water-coated ice spheroids, of Langleben and Gunn (1952) for larger water-coated spheres and the above-mentioned

* Despite the higher concentration (about 5 times) of snow particles caused by their lower fall velocities. This implies an increase in average particle mass at or about the level of melting.

shape effects for water, it seems reasonable to assume that the scattering and attenuation by water-coated ice spheroids of the size of wet snowflakes at radar wavelengths is similar in magnitude to that of spheroidal water drops of the same shape and size.

This is relevant to the bright band which always appears in radar pictures of steady rain. The bright band is due to a region of enhanced reflectivity a few hundred feet thick just below the 0°C isotherm. In the light of the foregoing discussion, the following mechanism is suggested. Snow falling through the freezing level will change from flat or needle-shaped particles or aggregates which scatter feebly, to similarly-shaped particles, which, owing to a water coating, scatter relatively strongly. As melting proceeds, they lose their extreme shapes and the shape factor decreases to unity. Because of the greater velocity of fall, the number of particles in unit volume also becomes less.

Evidence of shape effects in the bright band has been obtained from observations with cross-polarized radars (Browne and Robinson 1952; Hunter 1954). Non-spherical particles can scatter back power with different polarization from that transmitted. The topic is discussed more fully by Marshall, Hitschfeld and Gunn (1954).

Attenuation, like scattering, is proportional to the number of particles per unit volume, and therefore depends on the velocity of fall. Water spheroids absorb more strongly than water spheres, and coated ice spheroids presumably absorb like water spheroids. Therefore, absorption by melting snow will be many times that of the resulting rain. Although the bright band is comparatively thin, serious attenuation may take place when radar observations are made through it at shallow elevations.

6. Clouds

Most cloud droplets do not exceed 100 μ diameter, so that at centimetre and millimetre wavelengths the Rayleigh formulae (12) and (14) apply for droplets and Eqs. (3) and (6) for cloud.

(*a*) *Scattering from clouds*

Cloud droplets being about one-hundredth the diameter of raindrops, $\sum_1 D^6$ in cloud is about a factor 10^6 less than in rain. However, Eq. (3) for the received power has λ^4 in the denominator, so that it is possible to overcome some of the effect of the very small diameters by going to the shortest practicable wavelength. Radar sets working on wavelengths of 1·25 cm and around 9 mm are now being used for cloud observations (Plank, Atlas and Paulsen 1952; Swingle 1953). By directing the antenna vertically, the range r is made a minimum, and most clouds visible to the eye are detected by the receiver.

The radar signal is proportional to $Z = \sum_1 D^6$. To relate Z to other significant quantities such as liquid water content $M \left(= \frac{\pi}{6} \rho \sum_1 D^3 \right)$ would require the complete drop-size distribution. However, Atlas and Bartnoff (1953), after studying a number of such distributions actually observed in clouds, report that a knowledge of the median diameter only is sufficient for a fairly accurate relationship, since the shape of the distribution is similar for all clouds. Even without this knowledge, an approximate correlation between Z and M can be obtained (Atlas and Boucher 1952) because the median diameter shows a definite trend with M, and this can be incorporated in the empirical law

$$Z = 0{\cdot}0292\, M^{1{\cdot}82},$$

where M is in gm m^{-3}, and Z in mm^6 m^{-3}.

Donaldson *et al.* (1953) have recently made simultaneous measurements at 1·25 cm of scattered power and size distributions in fog. A preliminary analysis gives received powers 10 db below the theoretical values; the work is not yet sufficiently advanced to say whether or not this discrepancy can be accounted for by the measuring techniques.

(*b*) *Attenuation by clouds*

Since the cloud particles lie well within the Rayleigh region at all wavelengths in the centimetre band, Eq. (6) applies :

$$\text{attenuation (db km}^{-1}) = 0{\cdot}4343\,\frac{6\pi}{\lambda}\,\frac{M}{\rho}\,Im\,(-K) \qquad . \qquad . \qquad (6)$$

for one-way transmission. Thus attenuation by cloud depends on the liquid-water content and is independent of the particle-size distribution.

Values of the attenuation from water and ice clouds are given in Table 5 as a function of M for six wavelengths and three temperatures. For both ice clouds and water clouds $Q_s \ll Q_a$.

7. Concluding remarks

(*a*) *Refractive indices*

The measured values of the refractive index of water for various wavelengths and temperatures are quite adequate for microwave-precipitation work. The fact that the measured values fit the theoretical formulae so well (see, for example, Saxton and Lane 1946) allows interpolation and extrapolation about these values to be made with confidence.

The situation for ice is not so well established. There is agreement on the value of the real part of the refractive index and on its independence of temperature and wavelength in the microwave region. However, measured values of the imaginary part differ by a factor 2. Further determinations of this quantity are desirable.

(*b*) *Rain*

Drop-size distributions continue to be measured, both as an adjunct of scattering and attenuation measurements and independently, but they will probably not make much impression on the universal relationship of Eq. (16). Some understanding of the variation of drop-size distributions with time is now being acquired (Atlas and Plank 1953; Marshall 1953) which should make it easier to relate the radar signal to parameters measured at the ground.

Though Ryde (1945) evaluated the attenuation by rain at four wavelengths, and various temperatures, the work may bear repeating for the slightly changed values of the refractive index of water which have been reported since his computations were made, at least at 0°C, although the final results may not be very different.

One outstanding unsolved problem is the source of the 4-7 db difference between the actual signal received from both rain and snow and that predicted from the theory.

(*c*) *Snow*

Labrum (1952a) has observed the scattering from single melting spheroidal ice particles, but there has been little experimental work done on either scattering or attenuation by falling snow. There is a need for laboratory experiments to study the melting of natural snowflakes, both single crystals and aggregates. Point-to-point measurements of attenuation by melting snow would be particularly valuable.

(*d*) *Summary*

By way of summarizing, the back-scatter cross-sections per unit volume and the attenuation to be expected from rain, snow, water cloud and ice cloud for six wavelengths and various temperatures are shown in Tables 4 and 5.

When two or more of the elements listed in these Tables (and in Table 6, Appendix I) are present, the cross-sections and attenuations are additive. In a typical case, the attenuation in decibels will be the sum of the attenuations by rain, cloud and atmosphere.

TABLE 4. BACK-SCATTER CROSS-SECTION PER UNIT VOLUME $\sum_i \sigma$ IN $CM^2\ M^{-3}$

	T (°C)	$\lambda = 10$ cm	$\lambda = 5{\cdot}7$ cm	$\lambda = 3{\cdot}2$ cm	$\lambda = 1{\cdot}8$ cm	$\lambda = 1{\cdot}24$ cm	$\lambda = 0{\cdot}9$ cm
Rain* R in mm hr^{-1}	18	$5{\cdot}89 \times 10^{-6} R^{1{\cdot}5}$	$5{\cdot}59 \times 10^{-5} R^{1{\cdot}6}$	$8{\cdot}4 \times 10^{-4} R^{1{\cdot}56}$			$0{\cdot}16 R^{1{\cdot}23}$
Snow R in mm hr^{-1} of melted water	All Temps	$1{\cdot}21 \times 10^{-6} R^{1{\cdot}6}$	$1{\cdot}15 \times 10^{-5} R^{1{\cdot}6}$	$1{\cdot}15 \times 10^{-4} R^{1{\cdot}6}$	$1{\cdot}15 \times 10^{-3} R^{1{\cdot}6}$	Rayleigh approximations no longer applicable	
Water cloud M in gm m^{-3}	20	$8{\cdot}29 \times 10^{-4} M^{1{\cdot}82}$	$7{\cdot}86 \times 10^{-3} M^{1{\cdot}82}$	$7{\cdot}91 \times 10^{-2} M^{1{\cdot}82}$	$0{\cdot}785\ M^{1{\cdot}82}$	$3{\cdot}74\ M^{1{\cdot}82}$	$12{\cdot}4\ M^{1{\cdot}82}$
	10	$8{\cdot}32 \times 10^{-4} M^{1{\cdot}82}$	$7{\cdot}88 \times 10^{-3} M^{1{\cdot}82}$	$7{\cdot}91 \times 10^{-2} M^{1{\cdot}82}$	$0{\cdot}783\ M^{1{\cdot}82}$	$3{\cdot}46\ M^{1{\cdot}82}$	$12{\cdot}3\ M^{1{\cdot}82}$
	0	$8{\cdot}34 \times 10^{-4} M^{1{\cdot}82}$	$7{\cdot}89 \times 10^{-3} M^{1{\cdot}82}$	$7{\cdot}93 \times 10^{-2} M^{1{\cdot}82}$	$0{\cdot}781\ M^{1{\cdot}82}$	$3{\cdot}42\ M^{1{\cdot}82}$	$12{\cdot}1\ M^{1{\cdot}82}$
	− 8		$7{\cdot}92 \times 10^{-3} M^{1{\cdot}82}$ (extrapolated)	$7{\cdot}95 \times 10^{-2} M^{1{\cdot}82}$ (extrapolated)	$0{\cdot}778\ M^{1{\cdot}82}$ (extrapolated)	$3{\cdot}36\ M^{1{\cdot}82}$	$11{\cdot}6\ M^{1{\cdot}82}$

* The Rayleigh approximations have been used at 10 and 5·7 cm; at 3·2 and 0·9 empirical relations based on the curves of Fig. 3 are given.

TABLE 5. ATTENUATION DUE TO PRECIPITATION OR CLOUD (ONE-WAY) IN DB KM^{-1}

	T (°C)	$\lambda = 10$ cm	$\lambda = 5{\cdot}7$ cm	$\lambda = 3{\cdot}2$ cm	$\lambda = 1{\cdot}8$ cm	$\lambda = 1{\cdot}24$ cm	$\lambda = 0{\cdot}9$ cm
Rain* R in mm hr^{-1}	18**	$0{\cdot}0003 R^{1{\cdot}00}$	$0{\cdot}0022 R^{1{\cdot}17}$	$0{\cdot}0074 R^{1{\cdot}31}$	$0{\cdot}045 R^{1{\cdot}14}$	$0{\cdot}12 R^{1{\cdot}06}$	$0{\cdot}22 R^{1{\cdot}00}$
Snow*** R in mm hr^{-1} of melted water	0	$0{\cdot}035 \times 10^{-5} R^{1{\cdot}6} + 22{\cdot}0 \times 10^{-5} R$	$0{\cdot}33 \times 10^{-5} R^{1{\cdot}6} + 38{\cdot}5 \times 10^{-5} R$	$3{\cdot}3 \times 10^{-5} R^{1{\cdot}6} + 68{\cdot}6 \times 10^{-5} R$	$33{\cdot}2 \times 10^{-5} R^{1{\cdot}6} + 122 \times 10^{-5} R$		
	− 10	$0{\cdot}035 \times 10^{-5} R^{1{\cdot}6} + 7{\cdot}3 \times 10^{-5} R$	$0{\cdot}33 \times 10^{-5} R^{1{\cdot}6} + 12{\cdot}9 \times 10^{-5} R$	$3{\cdot}3 \times 10^{-5} R^{1{\cdot}6} + 22{\cdot}9 \times 10^{-5} R$	$33{\cdot}2 \times 10^{-5} R^{1{\cdot}6} + 40{\cdot}6 \times 10^{-5} R$	Rayleigh approximations no longer applicable	
	− 20	$0{\cdot}035 \times 10^{-5} R^{1{\cdot}6} + 5{\cdot}0 \times 10^{-5} R$	$0{\cdot}33 \times 10^{-5} R^{1{\cdot}6} + 8{\cdot}8 \times 10^{-5} R$	$3{\cdot}3 \times 10^{-5} R^{1{\cdot}6} + 15{\cdot}7 \times 10^{-5} R$	$33{\cdot}2 \times 10^{-5} R^{1{\cdot}6} + 28{\cdot}0 \times 10^{-5} R$		
Water cloud M in gm m^{-3}	20	$0{\cdot}39 \times 10^{-2} M$	$1{\cdot}36 \times 10^{-2} M$	$4{\cdot}83 \times 10^{-2} M$	$12{\cdot}8 \times 10^{-2} M$	$31{\cdot}1 \times 10^{-2} M$	$64{\cdot}7 \times 10^{-2} M$
	10	$0{\cdot}56 \times 10^{-2} M$	$1{\cdot}96 \times 10^{-2} M$	$6{\cdot}30 \times 10^{-2} M$	$17{\cdot}9 \times 10^{-2} M$	$40{\cdot}6 \times 10^{-2} M$	$68{\cdot}1 \times 10^{-2} M$
	0	$0{\cdot}90 \times 10^{-2} M$	$2{\cdot}72 \times 10^{-2} M$	$8{\cdot}58 \times 10^{-2} M$	$26{\cdot}7 \times 10^{-2} M$	$53{\cdot}2 \times 10^{-2} M$	$99 \times 10^{-2} M$
	− 8		$3{\cdot}4 \times 10^{-2} M$ (extrapolated)	$11{\cdot}2 \times 10^{-2} M$ (extrapolated)	$34 \times 10^{-2} M$ (extrapolated)	$68 \times 10^{-2} M$	$125 \times 10^{-2} M$
Ice cloud M in gm m^{-3}	0	$7{\cdot}87 \times 10^{-4} M$	$13{\cdot}8 \times 10^{-4} M$	$24{\cdot}6 \times 10^{-4} M$	$43{\cdot}6 \times 10^{-4} M$	$63{\cdot}5 \times 10^{-4} M$	$87{\cdot}4 \times 10^{-4} M$
	− 10	$2{\cdot}62 \times 10^{-4} M$	$4{\cdot}60 \times 10^{-4} M$	$8{\cdot}19 \times 10^{-4} M$	$14{\cdot}6 \times 10^{-4} M$	$21{\cdot}1 \times 10^{-4} M$	$29{\cdot}3 \times 10^{-4} M$
	− 20	$1{\cdot}80 \times 10^{-4} M$	$3{\cdot}16 \times 10^{-4} M$	$5{\cdot}63 \times 10^{-4} M$	$10{\cdot}0 \times 10^{-4} M$	$14{\cdot}5 \times 10^{-4} M$	$20{\cdot}0 \times 10^{-4} M$

* These are empirical relations. For the curves on which they are based see Fig. 5.

** The effect of temperature on attenuation by rain is discussed in Section 4 (c).

*** These values are obtained using the Rayleigh approximations, which are not valid for wavelengths less than about 1·5 cm. A value of $R = 10$ or $R^{1{\cdot}6} = 39$ is an upper limit for snowfall rates.

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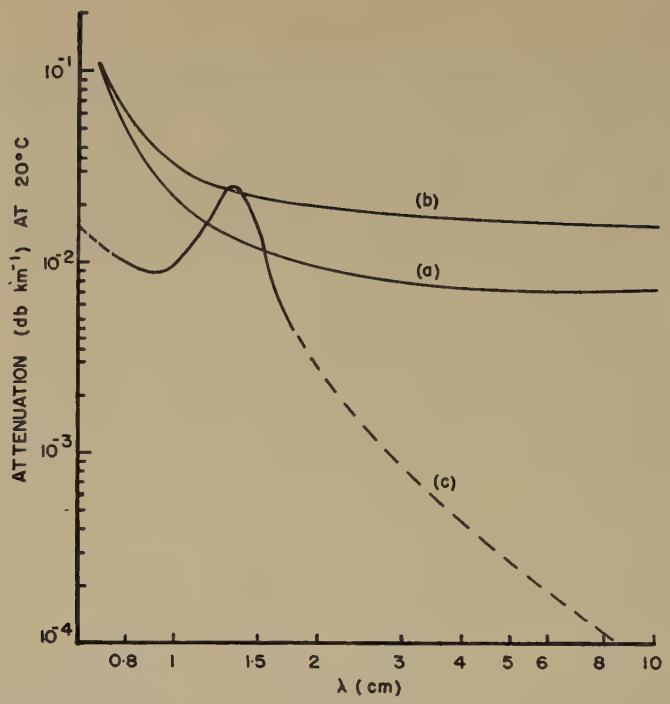

Figure 7. Atmospheric attenuation versus wavelength. (a) Attenuation due to O_2 in air at 1 atmos pressure and 293°K, using $\Delta\nu = 0{\cdot}039$ cm^{-1} for 5 mm lines and $\Delta\nu = 0{\cdot}02$ cm^{-1} for zero-frequency line. (b) As (a), using $\Delta\nu = 0{\cdot}039$ cm^{-1} for 5 mm lines and $\Delta\nu = 0{\cdot}05$ cm^{-1} for zero-frequency line. (c) Attenuation due to 1 g m^{-3} of water vapour in atmosphere : full line, measured (Becker and Autler 1946); dashed line, calculated. Attenuation is approximately proportional to water vapour content.

Appendix I

Attenuation by atmospheric gases

Eq. (7) does not take account of absorption by atmospheric gases. It has been shown by infra-red and, more recently, by microwave spectroscopy that oxygen molecules have a magnetic dipole moment and water-vapour molecules have an electric dipole moment. These give rise to lines in the absorption spectra which lie in the centimetre and millimetre region. This means that radar propagation is subject to attenuation by oxygen and water vapour in the atmosphere. Van Vleck (1947a, b) has discussed the problem in detail. He points out that these absorption lines are subject to the usual amount of 'pressure broadening.' In water vapour, under atmospheric conditions, the most important line has a line breadth (from resonance to half-intensity) of about 0·1 cm^{-1}. Since the wave-number of the line itself is only 0·74 cm^{-1} the line is relatively very broad; it has a resonant wavelength of 1·348 cm and its influence extends from about 1 cm to 2 cm wavelength. Additional effects are found due to other, much stronger, absorption lines centred near 0·2 cm wavelength.

Becker and Autler (1946) measured the damping of a large cavity resonator filled with air and water vapour. They used wavelengths between 0·75 cm and 1·7 cm. Their results are plotted as part of curve (c) in Fig. 7. The experiments were performed at 45°C, but have been corrected to 20°C in plotting the curve to bring it into line with the other curves. The dashed portions of curve (c) are calculations by Van Vleck from a theoretical formula with the constants chosen to fit Becker and Autler's results. Curve (c) is drawn for one gm m^{-3}, and attenuation is approximately proportional to water vapour content for the amounts encountered in the atmosphere. Dicke, Beringer, Kyhl and

Vane (1946) used a radiometer method to investigate absorption by atmospheric water vapour at 1·0 cm, 1·25 cm and 1·5 cm. Though the accuracy is not high, their results are consistent with those of Becker and Autler.

The situation for oxygen differs in two respects from that for water vapour; the most important absorption is caused by a number of closely-spaced lines instead of a single line and, in addition, oxygen has a line at zero frequency, while water vapour has not.

There are about 25 absorption lines between 0·45 and 0·56 cm wavelength; their positions were first predicted by Van Vleck and have been measured in pure oxygen at low pressures by Burkhalter, Anderson, Smith and Gordy (1950) and by Anderson, Smith and Gordy (1952). The later work included line breadth measurements which show values varying from line to line between $\Delta\nu = 0{\cdot}0319$ and $0{\cdot}0516$ cm^{-1}/atmos for pure oxygen*. An average value weighted for line intensity, and reduced by 18 per cent to allow for the difference in collision cross-section between pure oxygen and air, is $\Delta\nu = 0{\cdot}039$ cm^{-1}/atmos for air.

At atmospheric pressure this amount of broadening converts the line spectrum into an unresolved band of absorption. Lamont (1948) measured attenuation in air at wavelengths between 0·45 and 0·58 cm over a path of a few kilometres in the field. Between 0·5 and 0·53 cm the attenuation averaged 13 db km^{-1}; his results are in fair agreement with the theory extrapolated from low pressures.

Van Vleck's predictions include a ' line ' at zero frequency, and this accounts for most of the attenuation at wavelengths from 1 cm upwards**. The contribution of this ' line ' to the attenuation is independent of wavelength (for wavelengths shorter than 10 cm) and proportional to line breadth. No measurement appears to have been made above 0·6 cm specifically on oxygen, so that the line breadth for the zero-frequency line is not known with any certainty and the whole curve above 1 cm is in doubt. The radiometer measurements of Dicke *et al.* at 1·0 to 1·5 cm can be extrapolated back to zero water content, and indicate an absorption by oxygen corresponding to line breadth for zero frequency of $\Delta\nu = 0{\cdot}02$ cm^{-1}/atmos. The spread of the data, however, is large enough to make this figure very uncertain, and one can say little more than that the line breadth cannot be very much greater than this amount. Van Vleck (1947a) mentions a measurement by Mueller which indicates the same thing.

At the time that Van Vleck's calculations were made, it was believed that the line breadth constant $\Delta\nu$ was the same from line to line, though its value was not known. Accordingly, he assigned several alternative values to $\Delta\nu$ and the predicted attenuation for $\Delta\nu = 0{\cdot}02$ cm^{-1}/atmos and $\Delta\nu = 0{\cdot}05$ cm^{-1}/atmos (two likely values) were used by Ryde (1946) in his Fig. 1, curves (a) and (b).

Since so little is known, even at this date, about the zero-frequency line, in Fig. 7 we have used the same two values, viz. $\Delta\nu = 0{\cdot}02$ cm^{-1}/atmos, the most probable value, and $\Delta\nu = 0{\cdot}05$ cm^{-1}/atmos, which is to be taken as an upper limit, for the zero-frequency line together with the new, fairly reliable value $\Delta\nu = 0{\cdot}039$ for the 0·5 cm lines.

A laboratory experiment at any wavelength over 1 cm on dry air at atmospheric and at very low pressures, or a field experiment at any wavelength over 3 cm at a low humidity would determine the line-breadth constant and remove the uncertainty at wavelengths above the 0·5 cm band. However, since the attenuation appears to be not much greater than 0·01 db km^{-1} it is difficult to measure and at the same time usually unimportant in practice. Any application in which it becomes important to know it precisely will probably suggest a way of measuring it accurately.

* Added in proof – Artman (' Absorption of microwaves by oxygen in the millimetre wavelength region,' *Columbia Radiation Lab. Rep.*, 1953) finds that the line breadth is very nearly constant for these lines and that under atmospheric conditions it is 0·039 cm^{-1}/atmos.

** An alternative approach to this kind of absorption can be made by using Debye's theory of polar molecules in dielectrics.

The pressure and temperature dependence of attenuation are discussed by Van Vleck in his two papers. For a given quantity of water vapour, attenuation is proportional to p^{-1} and to $T^{-2}\,10^{-278/T}$ at the maximum of the resonance curve, to p and to $T^{-3}\,10^{-278/T}$ on the sides of the curves, and to p and to $T^{-3/2}$ well away from the resonance. Of course, if the air is kept saturated with water vapour while the temperature changes, the water vapour content itself will alter, and this additional factor must be taken into account. In these expressions T is in degrees Kelvin.

Table 6 shows attenuation by water vapour in air at various temperatures and wavelengths.

Pressure effect on atmospheric oxygen arises from increased pressure broadening and also from an increase in the number of molecules per unit volume. In the region of strong absorption, the attenuation is more or less independent of p and proportional to $T^{-3/2}$, but in the region from 0·7 to 10 cm wavelength it is proportional to p^2 and to $T^{-5/2}$.

Table 7 shows correction factors by which values read from curves (a) and (b) of Fig. 7 should be multiplied to obtain attenuation in conditions other than 20°C and 76 cm Hg.

TABLE 6. Water vapour attenuation (one way) in db km^{-1}

P, pressure in atmospheres; W, water vapour content in g m^{-3}.

	λ (cm)					
T (°C)	10·0	5·7	3·2	1·8	1·24	0·9
20	$0{\cdot}07 \times 10^{-3}\,PW$	$0{\cdot}24 \times 10^{-3}\,PW$	$0{\cdot}7 \times 10^{-3}\,PW$	$4{\cdot}3 \times 10^{-3}\,PW$*	$22{\cdot}0 \times 10^{-3}\,P^{-1}\,W$*	$9{\cdot}5 \times 10^{-3}\,PW$
0	$0{\cdot}08 \times 10^{-3}\,PW$	$0{\cdot}27 \times 10^{-3}\,PW$	$0{\cdot}8 \times 10^{-3}\,PW$	$4{\cdot}8 \times 10^{-3}\,PW$*	$23{\cdot}3 \times 10^{-3}\,P^{-1}\,W$*	$10{\cdot}4 \times 10^{-3}\,PW$
− 20	$0{\cdot}09 \times 10^{-3}\,PW$	$0{\cdot}30 \times 10^{-3}\,PW$	$0{\cdot}9 \times 10^{-3}\,PW$	$5{\cdot}0 \times 10^{-3}\,PW$*	$24{\cdot}6 \times 10^{-3}\,P^{-1}\,W$*	$11{\cdot}4 \times 10^{-3}\,PW$
− 40	$0{\cdot}10 \times 10^{-3}\,PW$	$0{\cdot}34 \times 10^{-3}\,PW$	$1{\cdot}0 \times 10^{-3}\,PW$	$5{\cdot}4 \times 10^{-3}\,PW$*	$26{\cdot}1 \times 10^{-3}\,P^{-1}\,W$*	$12{\cdot}6 \times 10^{-3}\,PW$

* The pressure dependences shown here are only approximate. In the neighbourhood of the 1·35 cm water vapour absorption line (say between 1·0 cm and 2·0 cm) no simple power law is accurate.

TABLE 7. Pressure and temperature correction for oxygen attenuation for wavelengths between 0·7 and 10·0 cm

T (°C)	Factor (P is pressure in atmospheres)
20	$1{\cdot}00\,P^2$
0	$1{\cdot}19\,P^2$
− 20	$1{\cdot}45\,P^2$
− 40	$1{\cdot}78\,P^2$

Appendix II

Errors noted in the literature of scattering and absorption by spheres

While consulting the published work on the scattering and absorption of electromagnetic waves by spheres, a number of typographical errors were noticed. It may serve a useful purpose to list them here, though this list does not claim to be complete.

Mie (1908). In the expansions of the Spherical Bessel functions, Eq. (27) on page 388 contains an error. The expression for $K_2(-x)$ should begin $-3/x^2$.

The three identities on page 437 are confusing because some of the symbols are printed in the wrong type; they can be clarified by referring to Eq. (93) on page 424. Later authors have used a different notation, but the relationships are fairly simple and are stated in Goldstein (1946), Eq. (39) on page 155.

Kerr (1951) states correctly the expansions of a_1, b_1, and b_2 on page 451, Eqs. (30a, b, c) (Eqs. (11a, b, c) in the present paper).

Goldstein (1946). On page 155, Eq. (38), the expression for b_1 should have a + sign before the ρ^2 term in the bracket; on page 171, it is given correctly.

Stratton (1941). P. 571, Eq. (40) has several errors. Goldstein (1946), page 155, has listed them in a footnote. He also mentions the following : a minus sign is missing in Stratton's Eqs. (26) for W_t and (29) and (31) for Q_t. At the top of the next page Q_s should equal Q_t. In Eq. (35) the denominator of the ρ^2 term should be $(2n + 2)(2n + 3)$, and in Eq. (36) there should be a minus sign.

Sinclair (1947) reviews various ways in which an incorrect factor 2 sometimes appears and explains the conditions under which the result $Q_t = 2\pi r^2$ for $\alpha \gg 1$ is correct.

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Properties of spectra of atmospheric turbulence at 100 metres

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Summary

The properties of spectra of vertical and horizontal turbulence and of their cross-spectra, at elevations near 100 m, are summarized ; a few spectra from lower levels are considered. The following tentative conclusions are drawn :

1. At frequencies of the order of 100 c/hr and above, the spectral intensity is proportional to the square of the mean wind speed and essentially independent of the radiation intensity. At lower frequencies, the ratio of spectral intensity to the square of the wind speed increases with increasing incoming radiation.

2. The intensity-ratio of the co-spectrum of vertical and horizontal velocity to the spectrum of vertical velocity increases with increasing wind shear. It also decreases with increasing frequency, n, approximately as n^{-1}.

3. At low frequencies, the spectral intensity of the horizontal velocity components is generally much larger than that of the vertical velocity. At high frequencies, the spectral intensities of all velocity components are of the same order of magnitude. At low frequencies, the ratio of vertical to horizontal spectral density increases with increasing radiation.

4. The quadrature spectra at a given frequency tend to be negative with large radiation, positive with little radiation. This indicates that eddies are low and wide with little radiation, and tall with much radiation.

5. The frequency at the maximum of the vertical velocity spectrum decreases with increasing height and increasing radiation intensity.

6. The ratio of turbulent to mean energy is somewhat larger for air with over-land trajectories than for air with oceanic trajectories.

1. Introduction

The classical Reynolds definitions of turbulence do not specify what scales of motion are important for the production of turbulent energy and flux. Power spectra and cross-spectra give the necessary information. Relatively few measurements of power spectra and cross-spectra of meteorological turbulence have been published. MacCready (1953) and Robinson (1953) obtained local power spectra of wind components close to the ground. The authors (1952 and 1953) presented some crude estimates of vertical-velocity spectra and of cross-spectra between vertical and horizontal wind components near 100 m. The present paper extends this work to more exact analysis of spectra and cross-spectra of wind components at three levels, and discusses the relation of these spectra to the mean atmospheric properties.

All estimates of spectra of meteorological turbulence contained in this paper and earlier publications are estimates of spectra at a point, as functions of frequency. Under the assumption, made by Taylor (1938), that the turbulence velocities are small compared to the mean wind, these local spectra can be converted into spectra as functions of wave number. This type of spectrum is of great interest to designers of airplanes and missiles and, at high frequencies, has been the subject of many theoretical discussions (see, for example, Priestley and Sheppard 1952, and MacCready 1953).

2. Instruments

All of the data for this study were obtained by means of instruments mounted on the 125-metre tower at the Brookhaven National Laboratory, Upton, L.I., New York. The laboratory is located near the centre of Long Island (see Fig. 1) in generally flat or slightly rolling countryside wooded by scrub pine and scrub oak some 10 m in average height. The tower itself, of open steel-framework construction, triangular in cross

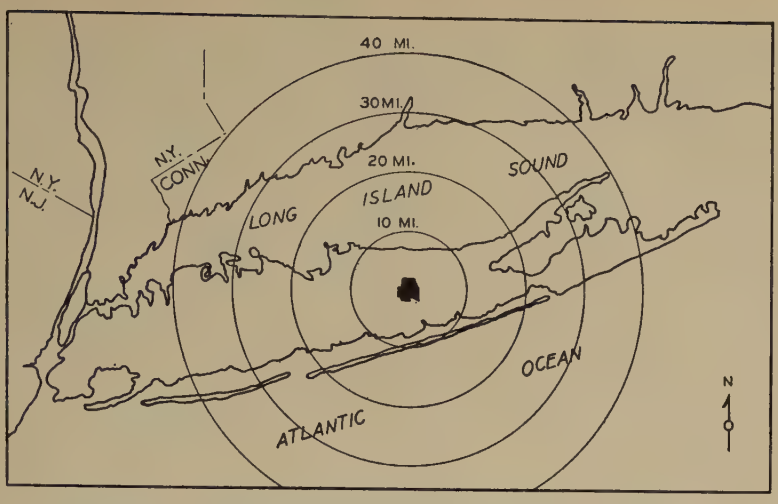

Figure 1. General location of Brookhaven National Laboratory.

section, is located on the margin of a clearing with several low buildings in the immediate vicinity. There are eight ' working levels ' on the tower, at elevations of 3 m, 6 m, 11 m, 23 m, 46 m, 91 m, 108 m, and 125 m. Each level is equipped with a counter-weighted tubular boom about 6 m in length, which can be pivoted through 315° and ' pulled in ' to a platform for the mounting or servicing of instruments.

Temperature measurements are made at all levels using Leeds and Northrup ' thermohms ' as the sensing elements. In addition, a separate system of matched thermohms at the 11 m, 46 m, and 125 m levels provides direct continuous measurements of the temperature difference between 125 m and 11 m, and between 125 m and 46 m on Leeds and Northrup Model R Micromax recorders.

Wind direction and velocity are measured on all but the lowest level by standard Bendix-Friez aerovanes. Recording of the traces is accomplished by Esterline-Angus twin recorders with a normal chart speed of 1 in. $(20\ \text{min})^{-1}$. In procuring the data for this study, the chart speed was increased to 1 in. $(20\ \text{sec})^{-1}$. A system of special aerovanes on the 11 m, 46 m, and 125 m levels gave direct measurements of velocity gradients, in conjunction with a unique wind-gradient recorder.

Three all-weather, remote-recording bivanes (one of which is described by Mazzarella 1952) provide measurements of the vertical deflection and horizontal direction of the wind. From wind-tunnel tests, Mazzarella has determined that the response characteristics of the bivanes and the aerovanes are nearly alike and are functions of the wind speed as indicated in Table 1 :

TABLE 1. Characteristics of instruments

Instrument	Damping constant (sec^{-1})	Free period (sec)
Aerovane	$0{\cdot}138\ \overline{V}$	$12{\cdot}28/\overline{V}$
Bivane vertical	$0{\cdot}112\ \overline{V}$	$10{\cdot}06/\overline{V}$
horizontal	$0{\cdot}141\ \overline{V}$	$10{\cdot}06/\overline{V}$

where $\overline{V}$ is the wind speed in m sec^{-1}. The wind-tunnel tests further show that the wind speed indicated by the aerovane is equal to the true three-dimensional wind speed

multipled by the cosine squared of the angle of attack. This factor becomes important in evaluating the orthogonal wind components at high wind speed and was taken into account in the appropriate cases by assuming the vertical deflection angle of the bivanes equal to the angle of attack of the aerovane. Radiation was measured by means of an Eppley pyrheliometer.

A few of the data presented in this paper were obtained from instruments of the Woods Hole Oceanographic Institution. This equipment consisted of a vertical vane, a thermistor anemometer compensated for temperature changes, and a thermistor thermometer, all mounted on a single frame. The frame was rotated on a vertical axis by a servomechanism which is activated by a small horizontal vane. The instruments have characteristic times and free periods of less than a second, so that the traces could be read every second. The assembly of these instruments before the construction of the servomechanism has been described by Parson and Bunker (1952).

From all Brookhaven records, 5-sec or 10-sec averages were estimated by eye from the traces, except for one period with extremely high wind speeds which made estimates possible every $3\frac{1}{2}$ seconds. The Woods Hole traces were read every second. Observations were extended over periods usually slightly exceeding one hour. Table 2 summarizes the conditions during the observation periods.

TABLE 2. CONDITIONS DURING OBSERVATION PERIODS

Period	Date	Time (GMT)	$(\overline{T}_1 - \overline{T}_3)$* °C	$(\overline{V}_3 - \overline{V}_1)$* m sec^{-1}	V m sec^{-1}	Wind direction	Radiation intensity cal cm^{-2} min^{-1}
A	31.8.51	1749-1909	1·7	2·7	7·2	SSW	1·1
B	31.8.51	1445-1550	1·6	1·1	3·7	SW	0·9
F	14·3·52	2030-2120	1·4	4·4	9·8	NW	0·6
G	24·7.52	1759-1850	2·5	3·5	8·7	NNW	1·3
H	1.9.52	1901-2010	0·6	8·4	15·3	SE	0·2
I	2.9.52	1903-2056	1·7	3·2	7·8	SSW	0·9
J_1	3.9.52	1616-1657	1·3	3·3	6·8	NW	0·5
J_2	3.9.52	1657-1750	1·7	3·8	8·2	NW	1·3
K	12.12.52	1600-1900	1·5	4·3	9·9	NW	0·6
K_1	12.12.52	1600-1700	1·5	4·3	10·3	NW	0·6
K_2	12·12.52	1800-1900	1·5	4·3	9·9	NW	0·6
L	5.12.52	1825-1925	0·6	7·1	12·8	ESE	0·1
P	15.1.53	1815-1915	1·2	4·2	9·1	SW	0·5
Z (108 m)	27.3.51	1639-1812	2·0	1·9	6·0	SSW	1·3
α	18.6.52	1245-1350	1·3	3·0	6·0	W	1·1
β	9.7.52	1410-1515	1·3	4·7	10·3	SE	0·3
γ	20.3.52	2010-2115	1·3	1·4	4·5	SSW	0·5
δ	10.3.52	1615-1725	1·4	1·8	7·0	SE	0·8
2	26.1.54	2000-2040	1·3	5·0	9·3	SSW	0·3
3	8.2.54	1450-1550	1·4	4·7	8·1	WSW	0·4

* Subscript 1 indicates the 11-m level, 3 indicates the 125-m level. All data refer to 91 m, except when otherwise indicated.

3. METHOD OF ANALYSIS

Generally, the analysis was limited to the spectra of vertical velocity and horizontal speed, as well as the cross-spectra between these variables. In several situations, however, wind components in particular directions were analyzed instead of the wind speed.

The method used for estimating the spectra was that recommended by Tukey (1950). It is based on the theorem that the cosine transform of the autocorrelation function of a time series is essentially equal to the spectrum of the original series. The reason why the spectrum is computed by way of the autocorrelation function rather than by direct harmonic analysis of the original record is this : direct harmonic analysis yields a number

of harmonics equal to half the number of observations. The amplitude of these harmonics oscillates wildly from one harmonic to the next. These oscillations are not reproducible from one time series to another which has basically the same statistical properties. We therefore need to compute a smoothed spectrum. The autocorrelation method, with number of lags small compared to the number of observations, yields a smoothed spectrum directly, with a great deal less numerical work than would be required for computing and smoothing the spectral estimate obtained by direct Fourier analysis.

The actual computation procedure recommended by Tukey is as follows :

First, mean lag products $\overline{x_i\, x_{i+L}}$ are formed of the variable x_i and the same variable after a time interval L. Secondly, the mean lag products for several consecutive values of L from 0 to m are subjected to a cosine transform. This step is followed by some smoothing by means of a weighted 3-term moving average (weights 0·23, 0·54, 0·23). In most of this study, the lag products were determined by standard IBM procedures; the cosine transforms and subsequent smoothing were done by hand computing. In periods *I*, *P*, 2 and 3 (Table 2), all the analysis was completed by the IAS electronic computer.

The choice of the number of lags, m, is critical. If too many lags are used, the numerical work is enormous. Further, sampling theory shows that the individual spectral estimates are quite uncertain. However, large m yields better resolution at low frequencies. The uncertainty of the spectra for large m can be decreased by averaging successive spectral estimates : but then, the major advantage of low-frequency resolution with large m is lost. In compromise, several different values of m were used for each spectrum.

When the spectrum varies rapidly with changing frequency, the method of spectrum analysis by way of the lag products and moving-average smoothing yields poor estimates. This is particularly serious for the horizontal speed spectra, where the spectral intensities may decrease rapidly with increasing frequencies. Occasionally, in fact, the spectral estimates at the high frequency end of the speed spectrum were negative. To remedy this situation, Tukey has recommended a further modification of the standard technique.

The original time series is first transformed to another series which has a spectrum not varying rapidly with frequency. The spectrum of the second series is then determined by the above procedure, and the spectrum of the original series found from it. If x_i represents the original series, the transformed series is defined by :

$$y_i = x_i - cx_{i-1} - (1 - c)\,\bar{x} \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

Here, c is a constant between 0 and 1, $\bar{x}$ the mean of x_i. The new time series has a spectrum in which the high frequencies are emphasized relative to the low frequencies. Given $S_y(n)$, the spectrum of y at frequency n, the spectrum of $(x - \bar{x})$ can readily be computed by :

$$S_x(n) = S_y(n)\,[1 + c^2 - 2c\cos(\pi\, n/m)]^{-1} \qquad . \qquad . \qquad (2)$$

In practice, c was generally taken as 0·75.

Whereas the ' spectrum ' measures the contributions of various frequencies to the variations of a single variable, the ' cross-spectra ' measure the contributions of the various frequencies to the product of two variables. There are two kinds of cross-spectra; co-spectra and quadrature spectra. Co-spectra analyze the product of two time series observed simultaneously. Thus, in particular, the co-spectrum of vertical and horizontal velocity components describes the contribution of various frequencies to a component of the stress, provided the variations in density can be neglected.

To understand the quadrature spectrum, consider each Fourier component of one of the variables shifted by a quarter-period relative to that of the other variable, and

the two then multiplied together. The quadrature spectrum then measures the contri bution of the various frequencies to this peculiar product. The potential significance of a quarter-period relative shift of the variables can be seen by considering the relation between dynamic height and southerly wind component at 700 mb. If a *co-spectrum* analysis were made of the two variables, it would be found that every harmonic of dynamic height would be about ninety degrees out of phase with the corresponding harmonic of wind component. Thus, the co-spectrum of height and southerly wind would be near *zero* even though the two variables are not independent. Yet the quadrature spectrum would be large.

There is a certain arbitrariness in the definition of the quadrature spectrum : either variable can lag behind the other by a quarter-period. In this paper, a quadrature spectrum is defined as positive where the vertical velocities lag a quarter-period behind the horizontal speeds or velocity components. In order to interpret the physical meaning of positive or negative quadrature spectra of vertical and horizontal velocities, consider a group of eddies (Fig. 2) moving across the paper from left to right.

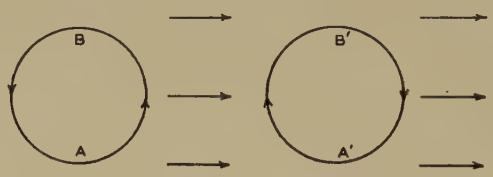

Figure 2. Eddy structure and quadrature spectrum

As the eddies pass a point A in the figure, upward vertical motion will be followed by excess speed, which in turn is followed by downward motion. Hence the horizontal excess speed and the vertical motion a quarter-period later have opposite signs. Therefore the quadrature spectrum, as defined above, is negative at A. At point B, near the top of the eddy, the sign of the horizontal speed excess is the same as that of the vertical velocity a quarter-period later. Hence, the quadrature spectrum is positive at B. Summarizing, quadrature spectra are negative near the bottom of eddies, positive near the top. The same distribution of quadrature spectra holds at points A′ and B′, so that the distribution of quadrature spectra is independent of the sense of the motion in the eddies.

The computation of cross spectra between two time series begins with the formation of mean lagged products *between* the two series, with lags both negative and positive. The co-spectrum is obtained by subjecting the means of negatively and positively lagged series to a cosine transform; the quadrature spectrum is based on one-half of the difference between positively and negatively lagged mean products. These differences are odd functions of the lags L, and are therefore analyzed by a sine transform.

Finally, the ' coherence ' between two variables x and y is defined by :

$$\mathrm{Coh}_{xy}(n) = [C^2_{xy}(n) + Q^2_{xy}(n)]\,[S_x(n)\,S_y(n)]^{-1} \quad . \quad . \quad (3)$$

where $S_x(n)$ is the spectral density of the variable x at frequency n, and $S_y(n)$ that of variable y at the same frequency. $C_{xy}(n)$ and $Q_{xy}(n)$ represent the co-spectrum and quadrature spectrum of the two variables, respectively. The coherence is analogous to the square of a correlation coefficient; it shows what fraction of the variance of one variable is accounted for by its association with the other. The coherence differs from the square of an ordinary correlation coefficient in that it varies with frequency.

4. General properties of spectra, cross-spectra and coherence

Spectra of vertical velocities were computed for all the 18 periods. In addition, spectra of horizontal wind speed, or the wind component in the direction of the mean speed, were found in all cases except period A, where the spectrum of the west-east wind component was analyzed. Computation in two cases indicated that the difference between the spectrum of wind speed and the wind component in the mean wind direction is negligible. Finally, cross-spectra were obtained between horizontal and vertical wind components in cases which are designated by roman letters and numerals.

Since meteorological turbulence covers a wide range of frequencies, a logarithmic frequency scale was judged appropriate. In order that the area between two frequencies under the spectral curve represent the variance contributed in this frequency interval, all spectral estimates had to be multiplied by the frequency. The term ' logarithmic ' spectrum is used for the product of spectral density and frequency.

Sixteen of the periods occurred when the stratification up to 125 m was super-adiabatic. The remaining two observations, in periods H and L, represent stable conditions with unusually high wind speeds.

For lack of space, the individual results are not given here. However, the estimates of logarithmic spectra and cross-spectra as well as their fiducial limits are presented in the Final Report of Project AF 19(604)-166, Air Force Cambridge (USA) Laboratory.

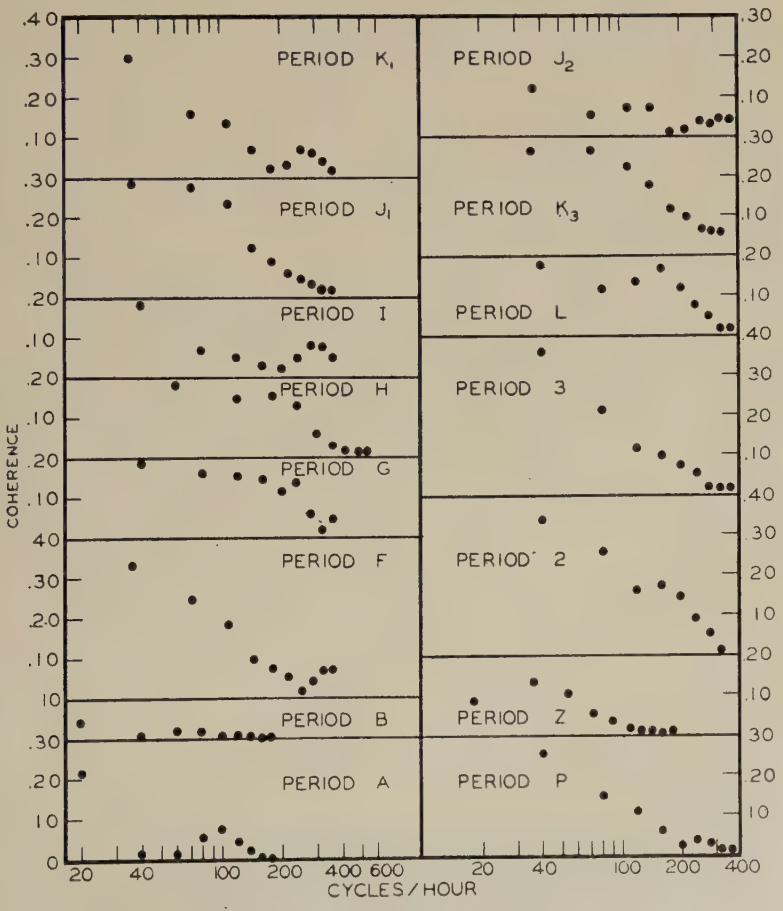

Figure 3. Coherence of vertical and horizontal velocity components.

A number of qualitative conclusions can be drawn by inspection of the various spectra and co-spectra :

(*a*) Within the frequency range analyzed, the spectra and cross-spectra seem to be remarkably continuous. In a few of the spectra, the spectral estimates at some frequencies dip below the estimates at frequencies to either side. Such gaps tend to be on the boundary of statistical significance. However, it will be seen in Section 6 that there is some evidence of the existence of gaps in the spectra outside the frequency range observed.

(*b*) The spectra of the horizontal velocity components, and the co-spectra of vertical and horizontal velocity components decrease more rapidly with increasing frequencies than the vertical-velocity spectra. At low frequencies, there is more energy in horizontal turbulence than vertical turbulence, except in cases of extreme instability.

(*c*) Stable stratification suppresses the energy at relatively low frequencies, that is, at frequencies less than 100 c/hr. As a result, the centre of gravity of the spectra shifts toward higher frequencies with increasing stability. These properties are discussed in more detail in Section 6.

(*d*) Quadrature spectra at low frequencies (less than 50 c/hr) near 100 m tend to be negative near the times of maximum convection, that is, on summer days near noon. Quadrature spectra during times of less convection are near zero at low frequencies and positive at medium frequencies (150 c/hr).

Fig. 3 summarizes the coherence between vertical and horizontal velocity components for various periods. The coherence decreases slowly with increasing frequency. The coherence is least during periods of smallest vertical wind shear, as might be expected from mixing-length arguments.

5. Effect of wind speed on spectra

Other things being equal, increasing wind speed would be expected to increase the frequency of the oscillations of any atmospheric parameter measured at a point. We may assume that this effect of translation by the wind increases the frequencies proportionally to the mean wind speed. Hence, we introduce the reduced frequency $f = \frac{nz}{\overline{V}}$ which should be independent of this effect. The height z has been inserted in order to make the reduced frequency nondimensional. If the mean wind speed is large enough compared with the turbulent deviations, the reduced frequency measures approximately the ratio of the height above the ground to the wave length.

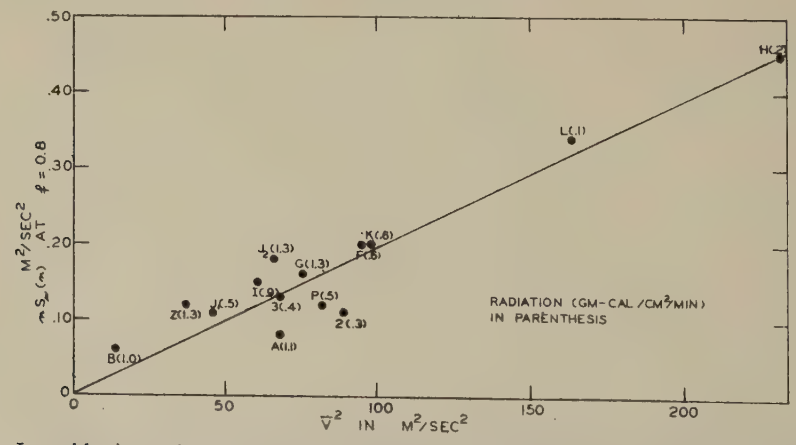

Figure 4. Logarithmic vertical velocity spectrum at $f = 0{\cdot}8$ as function of square of mean wind speed. Radiation entered in parentheses.

The second effect of the wind speed is to increase the intensity of mechanical turbulence. Fig. 4 shows the logarithmic vertical spectral density at the relatively high reduced frequency of 0·8 plotted as function of the square of the wind speed. Apparently, a linear relation exists. Further, the relation seems to be independent of the short-wave radiation, which is also entered in the figure. As will be seen in the next section, the effect of varying the radiation makes itself felt at lower frequencies only. The dependence of the co-spectrum and the spectrum of horizontal wind speed on the mean speed of the wind is similar. One might have expected a dependence of the spectral and co-spectral densities on the square of the mean wind speed, since they represent an analysis of the variance or co-variance of wind components, both of which are quadratic quantities.

In order to study other properties of the spectra, all logarithmic spectral intensities were divided by the square of the mean wind speed, thus forming a non-dimensional ratio which should be independent of the mean wind speed, at least at high frequencies.

6. Effect of convection on the spectra

The importance of convection can be judged by relating the observed spectra and cross-spectra to such variables as the lapse rate, or the radiation received during the period of observation. Neither of these parameters is a satisfactory measure of convective activity. The lapse rate may remain constant while the convective activity increases. This happened, for example, on 31 August 1951, when observations were made both in the morning and the afternoon. The lapse rate did not change, but the radiation increased, and the spectrum altered in a manner which will be shown to be characteristic of increasing convective activity. Nor is the radiation an ideal measure of convection : for the strength of convection and height of the convective layer will also depend on the radiation prior to the time of observation. For example, relatively much radiation was received (1·1 cal cm^{-2} min^{-1}) at 13 hr on 18 June 1952, (period α), but the convective cells were not yet well developed. Nevertheless, the radiation amount at the time of observation was judged to be the best of the available parameters to characterize the state of convection. Probably, a measure such as the height of the unstable layer would have been a better measure, if it had been available.

In Fig. 5, the reduced frequencies at the maxima of the logarithmic vertical velocity spectra determined by eye are plotted as function of the radiation amounts given in Table 2. The figure shows that the frequency of the maxima increases with decreasing radiation. This must be interpreted to indicate that increasing convective activity is associated with greater relative importance of low-frequency turbulence.

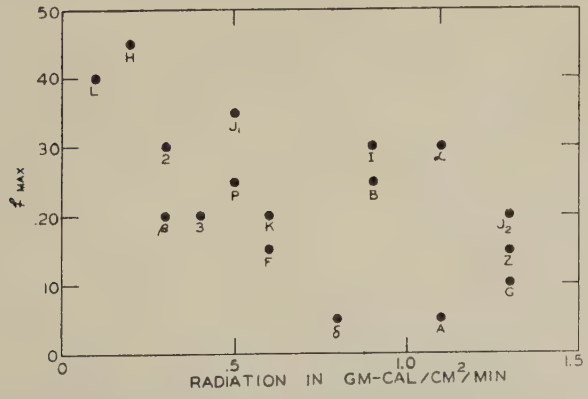

Figure 5. Reduced frequency of the maximum of the logarithmic vertical-velocity spectra as function of radiation intensity.

Fig. 5 suggests a grouping of the observations according to radiation amounts into three classes. Average logarithmic spectra were computed for these classes. Since the total of vertical motion energy is apparently proportional to the square of the mean wind speed at the level of observation, the average spectral estimates for each class were divided by the square of the mean wind speed for each class. Fig. 6 shows these composite spectra. The figure suggests this interpretation : at high frequencies, the spectra are independent of radiation. Hence, at these frequencies, *mechanical* turbulence is likely to predominate. On the other hand, the energy at lower frequencies ($f < 0{\cdot}6$) increases with increasing radiation. Therefore convection must play an important part in producing eddies with periods of the order of 30 sec and larger.

TABLE 3. DEFINITION OF RADIATION CLASSES

Class	Radiation (cal cm^{-2} min)$^{-1}$	Periods
Group (a)	$\geqslant 1{\cdot}0$	A, G, I, J_1, P, Z, α, 1, 3
Group (b)	0·3-0·9	F, J_1, K, β, γ, δ
Group (c)	$\leqslant 0{\cdot}2$	L, H

Fig. 7 shows the composite logarithmic spectra of horizontal speed or components in the mean wind direction, again split into three classes. As before, all spectra are divided by the square of the mean wind speed. Apparently, the separation due to convection occurs at lower frequencies than for the vertical-velocity spectra. Again, at low frequencies, the greatest energy goes with largest radiation amounts.

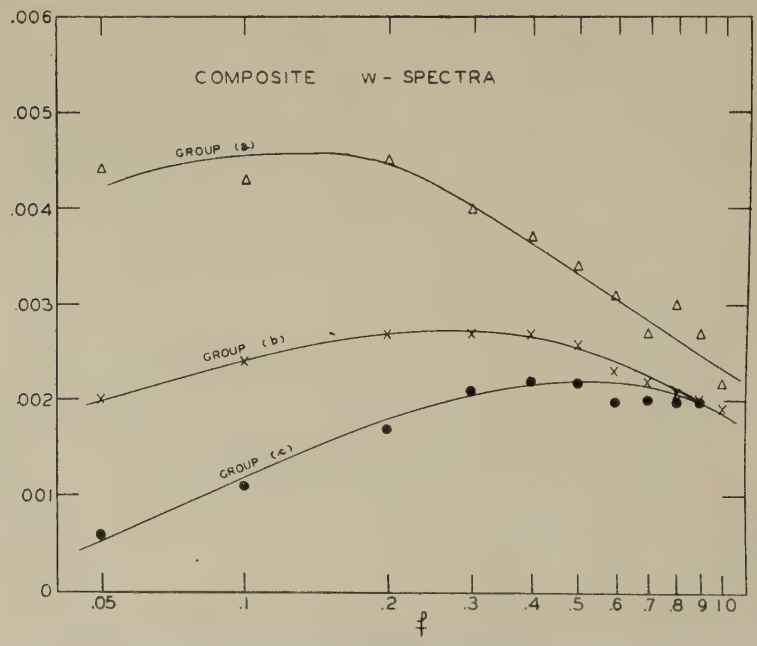

Figure 6. Composite logarithmic spectra of vertical velocities, divided by square of wind speed, grouped according to radiation amounts.

Since the primary maximum of the *complete* spectrum of energy of horizontal motion is very probably located near reduced frequencies of the order of $f = 5 \times 10^{-5}$, a gap is expected in the spectra of horizontal velocities at frequencies between $f = 0{\cdot}1$ and perhaps $f = 0{\cdot}001$, at least under stable conditions. The existence of such a gap can be proved only by longer records. If it is real, it would be extremely important for diffusion experiments, for then averages over a few minutes would be stable when the lapse rate is less than adiabatic.

Composite logarithmic *co-spectra* for the three groups of radiation classes were also computed. Their behaviour is similar to that of the horizontal spectra.

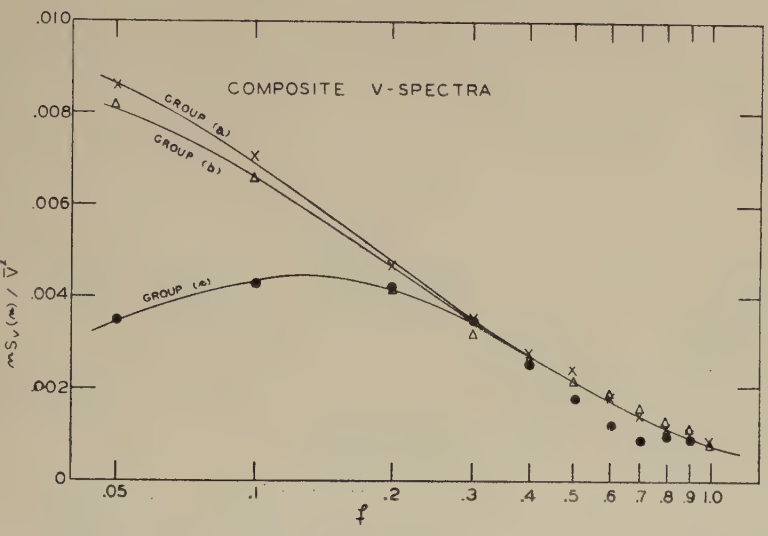

Figure 7. Composite logarithmic spectra of horizontal wind components, divided by the square of wind speed.

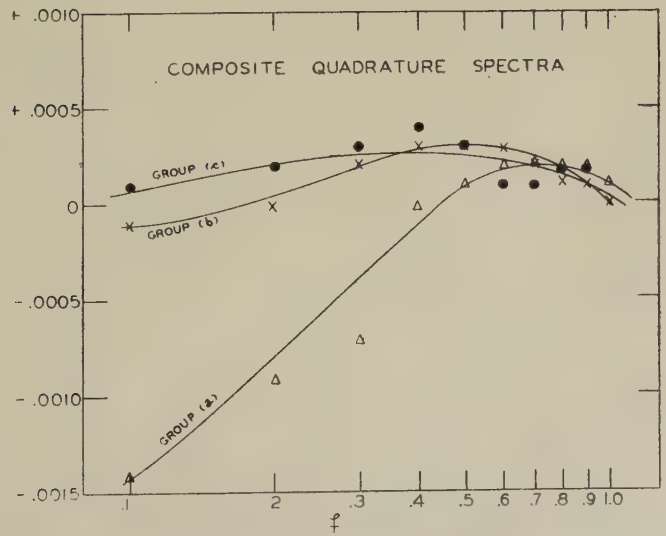

Figure 8. Composite logarithmic quadrature spectra divided by square of wind speed.

Fig. 8 shows composite logarithmic quadrature spectra for the three radiation classes. The behaviour of medium and large-radiation spectra differs significantly : when the radiation amounts were large, the largest eddies were much higher than the station height. When the radiation amounts were smaller, the largest eddies probably had their centres near the height of observations, whereas most smaller eddies were centred below 100 m.

Of special interest are the quadrature spectra during the three periods when simultaneous observations were available at several levels. In these cases (all of which belonged to the medium-radiation group) the quadrature spectra of the larger eddies were negative at 23 m, positive at 91 m, indicating that the 23 m level was in the lower portion of the eddies, but the 91-m level was near the top.

To sum up, low-frequency eddies during maximum convection are taller relative to their breadth than those at other times. This result is confirmed by two additional observations : (*a*) Gifford (1953) observed a similar difference between eddy structure during stable and unstable stratification by tracking no-lift balloons, and (*b*) the ratio of the spectral densities of vertical to horizontal velocity components at $f = 0{\cdot}3$ increases with increasing radiation (Fig. 9).

7. Behaviour of the spectrum at high frequencies

A large portion of the spectrum of an atmospheric velocity component falls into the so-called ' inertial sub-range.' This is a range of frequencies in which energy enters the eddies only by decay of larger eddies, and is lost only to smaller eddies. In this region, turbulence is not influenced by molecular viscosity, since the direct viscous dissipation of eddy energy can be neglected. Presumably, the largest eddies in the sub-range have wavelengths somewhat smaller than the height above the ground (Taylor 1952), whereas the smallest have wavelengths large compared to 0·2 cm. (Batchelor 1953). This

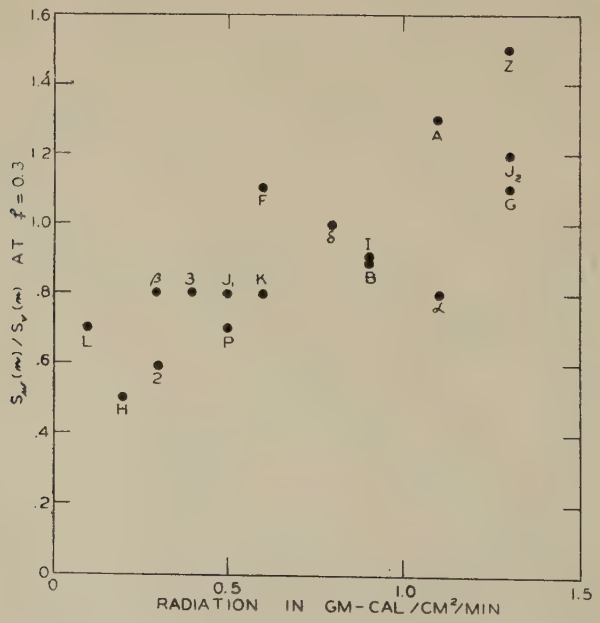

Figure 9. Ratio of vertical to horizontal spectral density at $f = 0{\cdot}3$ as function of radiation.

means that the inertial sub-range should exist above reduced frequencies of about $f = 1$. This frequency lies near the high-frequency limit of the observations, so that it is not possible to test the properties of the inertial sub-range rigorously. Nevertheless, an examination of the high-frequency end of the spectra might throw some light on the nature of the inertial sub-range. According to the Heisenberg-Kolmogoroff theory, the space spectra of the velocity components decrease with wave number, k, as $k^{-5/3}$. It is not certain whether the local *time* spectrum should depend on frequency in the same manner as the space spectrum depends on wave number. One might expect a $n^{-5/3}$ law if the wind speed was large compared to the eddy velocities, since in that case the time spectrum as a function of frequency would have the same characteristics as the space spectrum as function of wave number. This hypothesis underlies G. I. Taylor's discussion of the turbulence spectrum (1938), and has recently been verified in the atmosphere by MacCready (1953). Robinson (1953) prefers a dimensional argument independent of the space spectrum which leads to the conclusion that the spectra are proportional to n^{-2}. He finds that at frequencies above 4 c/sec, at 1·5 m, the local spectra are indeed proportional to n^{-2}. Evidence from airplane gust measurement is unable to distinguish between the k^{-2} and the $k^{-5/3}$ laws.

Near $f = 1$, the *horizontal* velocity spectra of the present set of observations generally indicate a variation of spectrum with frequency according to a power law with a power slightly more negative than -2. In fact the average power, derived from lines of log $S_v(n)$ as a function of log n, averaged $-2{\cdot}1$ for 16 spectra under unstable conditions. However, this figure by no means represents conclusive proof that the n^{-2} law is preferable to the $n^{-5/3}$ law, particularly since not all of the observations fell into the inertial sub-range.

In the case of the vertical velocities, the rapid decrease of spectrum with increasing frequency starts only at the highest frequencies analyzed so that the power of the spectrum-frequency law could not be computed with sufficient accuracy to distinguish between n^{-2} and $n^{-5/3}$. Thus, no clear conclusion can be drawn as to whether local power spectra vary as $n^{-5/3}$ or n^{-2} in the inertial sub-range.

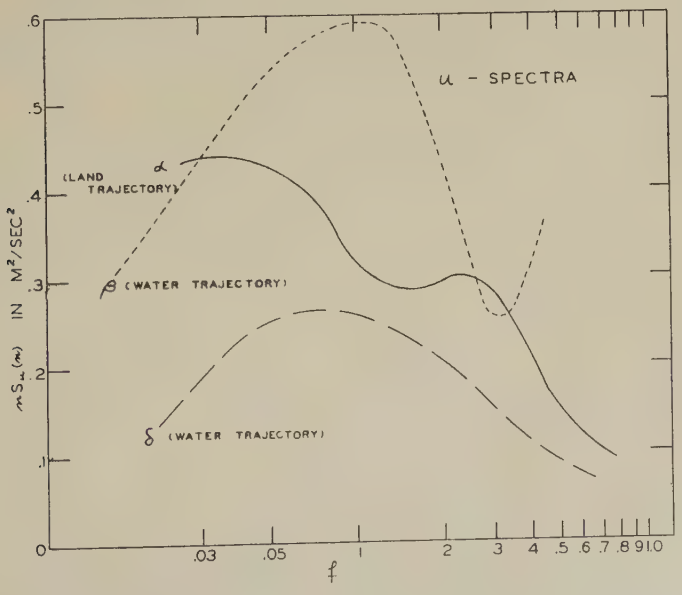

Figure 10. Spectra of velocity component in mean wind direction with different trajectories.

8. Effect of trajectory on the spectrum

Presumably the energy transformed from the mean motion into eddy energy, and manifest as 'mechanical' turbulence, would be reflected in the energy spectrum, especially in a frequency range which would depend on the size of the roughness elements over which the air is moving. The turbulence structure of an airmass moving from a comparatively smooth surface, such as the Atlantic Ocean, should be modified after passing over the coastline by the larger roughness elements represented by the Island surface (McCormick and Singer 1954). The modification should begin in the layers nearest the ground and propagate upward by the normal diffusing processes with increasing distance from the shore. If the modification has not reached the 91-m level when the air-mass reaches the tower, a comparative deficiency of energy in the air from an over-water path would be expected. The deficiency should be most evident in the frequency range which is sensitive to mechanical turbulence, that is at frequencies above $f = 0{\cdot}4$.

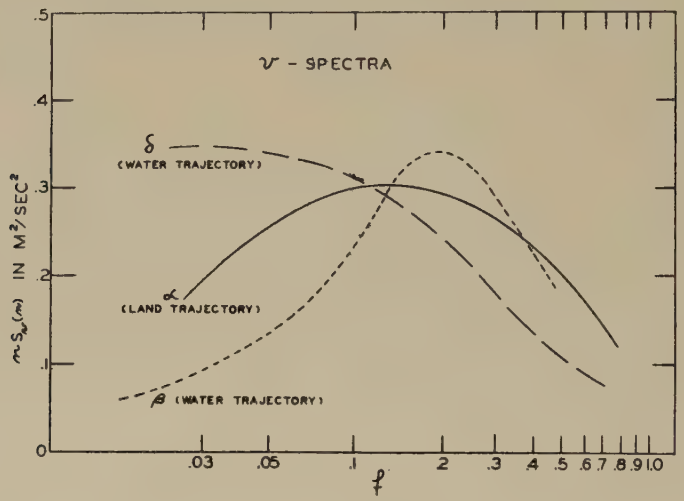

Figure 11. Spectra of horizontal cross-wind component with different trajectories.

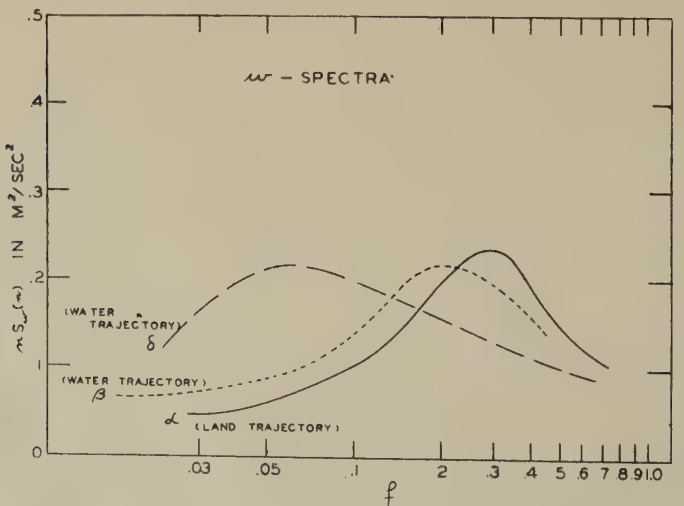

Figure 12. Spectra of vertical velocity with different trajectories.

Lines of best fit drawn by eye to the spectral estimates of cases α, β, and δ, are shown in Figs. 10 to 12. The trajectories of the air-masses sampled in each run were apparently clear-cut : β and δ were from over the Atlantic to the south-southeast, while α was over the mainland and the Island to the west. Since the lapse rates in the lower 125 m were identical and the total eddy energies almost equal, these spectra were expected to differ from each other mainly because of the different history of the air.

The spectra of the velocity components in the direction of the mean wind are shown in Fig. 10. These spectra are considerably dissimilar in shape and in the position of the maximum, but they do not indicate, in either an absolute or a relative sense, a tendency for a deficiency of energy at the higher frequencies for the over-water cases β and δ. This conclusion is also borne out by the comparisons of the cross-wind and vertical spectra, Figs. 11 and 12. In fact, the difference in the nature of the spectra between the two over-water cases is the most obvious anomaly and shows the apparent relative unimportance of the trajectories for the spectra in these cases.

The effect of possible influences of differences of the trajectory on turbulence spectra was further studied by the construction of composite spectra for observations with land trajectories on the one hand and ocean trajectories on the other. Stable periods were omitted, and all spectral densities were divided by the square of the wind speeds. Finally, horizontal and vertical energies were averaged together. The average lapse-rate and radiation amount for each composite spectrum was the same, so that the effects of wind speed and convective activity were presumably eliminated. Little difference was noted between the shapes of the spectra but the total turbulent energy relative to the energy of the mean motion was 28 per cent larger for over-land than over-ocean trajectories. But even this difference is not statistically ' significant.' In conclusion, it is quite likely that the eight miles of over-land travel between the ocean and the tower were sufficient to impose land characteristics on all turbulence spectra.

9. The ratio of co-spectra and vertical-velocity spectra

The co-spectra decrease with increasing frequency much more rapidly than the spectra. The authors showed by means of a mixing-length argument (1952) that the ratio of the co-spectrum $C_{Vw}(n)$ to the spectrum of vertical velocity $S_w(n)$ should be given by :

$$R(n) = -C_{Vw}(n)/S_w(n) = c\, n^{-1} \frac{\partial \overline{V}}{\partial z} \quad . \quad . \quad . \quad (4)$$

where c is an undetermined constant. From observations of the first 12 periods of Table 4, the following relation was determined by least squares :

$$R(n) = 0{\cdot}25\, n^{-1} (\overline{V}_3 - \overline{V}_1)(z_3 - z_1)^{-1} \quad . \quad . \quad . \quad (5)$$

It is clear that Eq. (5) is not quite the equivalent of Eq. (4) in that $(\overline{V}_3 - \overline{V}_1)(z_3 - z_1)^{-1}$ is systematically larger than $\frac{\partial \overline{V}}{\partial z}$ near 100 m. Nevertheless, the ratio $(\overline{V}_3 - \overline{V}_1)(z_3 - z_1)^{-1}$ is likely to be proportional to $\frac{\partial \overline{V}}{\partial z}$ so that Eq. (5) is consistent with Eq. (4). Table 4 shows the observed co-spectrum-spectrum ratios and those computed from Eq. (5). This table not only includes data for the periods used to determine the constant in Eq. (5), but also four additional periods for testing purposes. Apparently, theory and observations are in satisfactory agreement. The correlation coefficient is 0·86.

TABLE 4. COMPARISON OF OBSERVED AND COMPUTED RATIO OF $C_{Vw}(n)$ AND $S_w(n)$.

Period	60 c/hr		120 c/hr		180 c/hr	
	Observed	Computed	Observed	Computed	Observed	Computed
A	0·11	0·35	0·17	0·18	− 0·05	0·12
B	0·14	0·14	− 0·02	0·07	− 0·01	0·05
F	0·80	0·57	0·41	0·29	0·19	0·19
G	0·39	0·45	0·48	0·23	0·19	0·15
H	0·91	1·09	0·66	0·54	0·54	0·36
J_1	0·68	0·43	0·34	0·21	0·18	0·14
J_2	0·27	0·49	0·20	0·25	0·04	0·16
K_1	0·68	0·56	0·33	0·28	0·04	0·19
L	0·64	0·92	0·46	0·46	0·39	0·31
Z	0·21	0·25	0·06	0·12	− 0·04	0·08
I	0·42	0·41	0·20	0·21	0·13	0·14
P	0·81	0·55	0·36	0·27	0·19	0·18
2	0·88	0·65	0·47	0·32	0·33	0·22
3	0·67	0·61	0·29	0·30	0·06	0·20

Observations at lower levels indicate that the simple linear relation for the co-spectrum-spectrum ratio may give too large ratios for large values of $n^{-1}\frac{\partial \bar{V}}{\partial z}$. Fig. 11 shows the ratio $R(n)$ plotted as function of $n^{-1}\frac{\partial \bar{V}}{\partial z}$, for periods 2 and 3, at frequencies of 60, 120 and 180 c/hr, at heights of 23 m, 46 m, and 91 m. In these cases, $\frac{\partial \bar{V}}{\partial z}$ was determined from smoothed observations of mean wind speed at six levels from 11 m to 125 m.

Fig. 13 indicates that a linear relation between $R(n)$ and $\frac{\partial \bar{V}}{\partial z}n^{-1}$ holds for values of $\frac{\partial \bar{V}}{\partial z}n^{-1} < 2$, but that this linear relation over-estimates $R(n)$ for values of $\frac{\partial \bar{V}}{\partial z}n^{-1} > 2$. Nevertheless, it seems likely that within the range of the observations, $R(n)$ is well determined by $\frac{\partial \bar{V}}{\partial z}n^{-1}$.

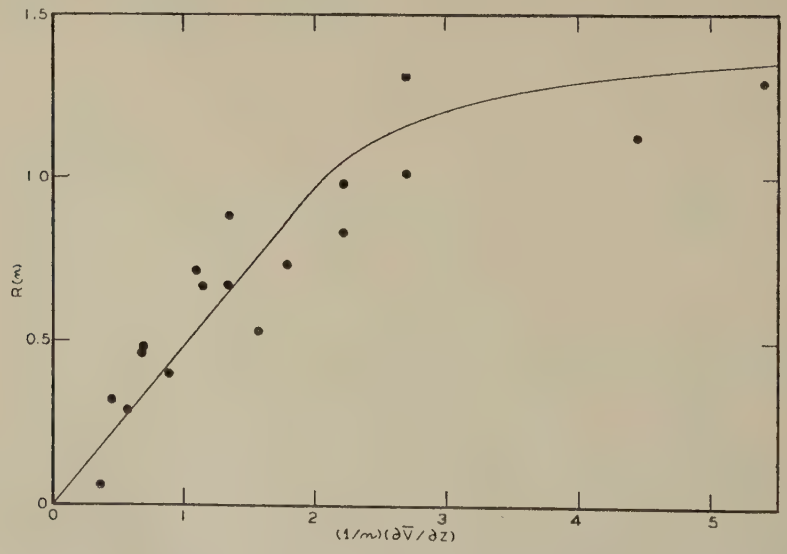

Figure 13. Co-spectrum-spectrum ratio as function of $n^{-1}\frac{\partial \bar{V}}{\partial z}$ for periods 2 and 3

10. Variation of spectra with height

Figs. 14 to 16 show the vertical-velocity spectra at various heights for periods B, 2 and 3. Periods B and 3 were characterized by superadiabatic lapse-rates whereas the lapse-rate in period 2 was unstable between 11 and 46 m and stable between 46 m and 125 m. It is remarkable that the maximum of the logarithmic spectra occurs at essentially the same value of f in the two superadiabatic cases at all heights, and also in the unstable portion of period 2. In other words, the frequency of the maximum of the vertical-velocity spectrum for simultaneous observations at different heights under unstable conditions appears to be proportional to $\overline{V}z^{-1}$ for the three periods studied here. There are indications that this proportionality may hold down to heights as low as 1 m. At least, the order of magnitude of the maximum of the spectrum agrees with that found by Robinson (1953). The relation is due to the limiting effect of the ground on the vertical extent of the eddies.

Not only the position of the maximum, but also the general shape of the spectrum as function of f, appears to be the same at all levels. In 1952, the authors proposed a working hypothesis for the vertical-velocity spectrum, which was to describe especially the effect of the ground. The equation suggested for the *normalized* spectrum was :

$$S_w(x)\,dx = 2x\,(1+x)^{-3}\,dx \quad . \quad . \quad . \quad . \quad (6)$$

Here x was originally defined by $x = \epsilon zn/\sigma_w$ where ϵ was assumed to be a parameter independent of height under superadiabatic conditions, and σ_w is the standard deviation of the vertical velocities. It now seems desirable to modify the definition of x to : $x = \dfrac{\gamma zn}{\overline{V}}$ where γ is another parameter invariant with height in an unstable atmosphere. An advantage of the re-definition of x is that Eq. (6) now depends only on variables usually measured. This re-definition is not merely a superficial change in the original working hypothesis, although the resulting expressions look similar. The change rather implies that the original hypothesis should be tested on the Lagrangian time spectrum rather than on the local time spectrum. The derivation of the original hypothesis made no distinction between the two types of spectrum; but its application to the local spectrum

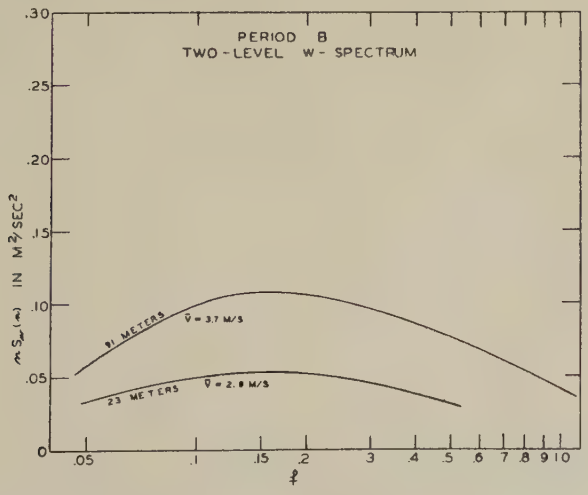

Figure 14. Logarithmic vertical-velocity spectrum at 23 m and 91 m in period B

was probably erroneous. The modified working hypothesis can be derived in a manner analogous to the original hypothesis : it is essentially an interpolation formula between the assumed behaviour of the vertical velocities at low frequencies due to the influence of the ground, and the assumed behaviour at high frequencies in the inertial sub-range.

Eq. (6) generally gives approximately the right degree of spread of the spectra, but some of the spectra have larger dispersion and some less. No simple equation of the type of Eq. (6) would fit all vertical-velocity spectra observed in an atmosphere with unstable stratification.

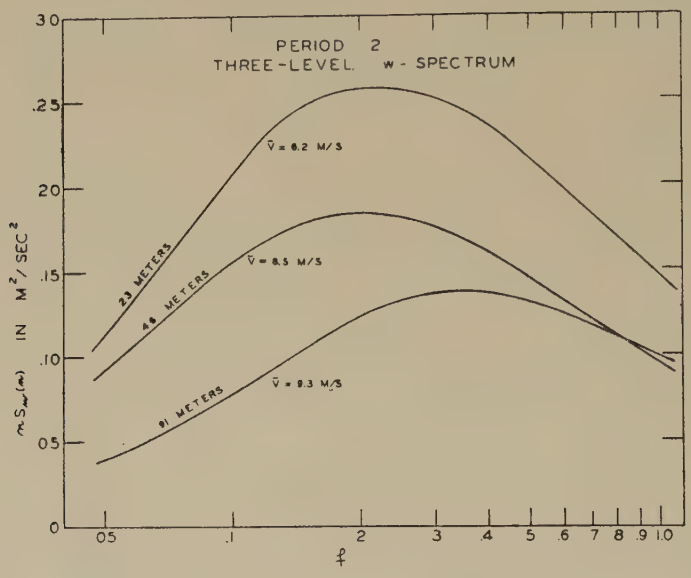

Figure 15. Logarithmic vertical-velocity spectrum at 23 m, 46 m, and 91 m, in period 2.

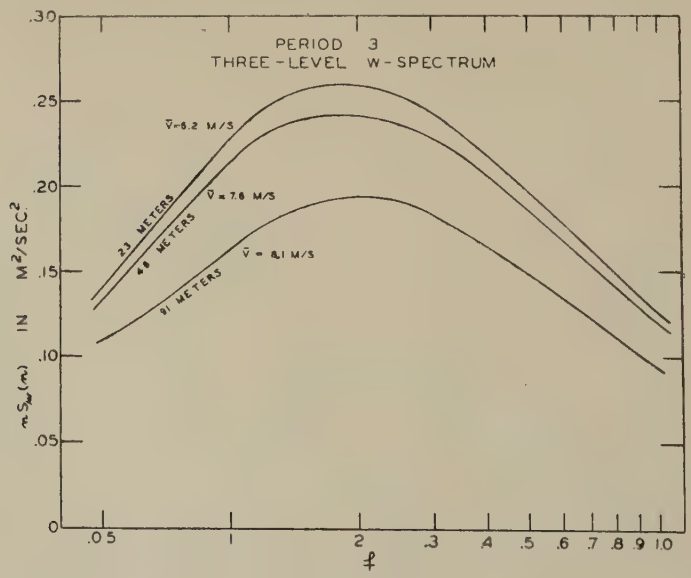

Figure 16. Logarithmic vertical-velocity spectrum at 23 m, 46m, and 91 m in period 3.

At high frequencies, Eq. (6) varies as n^{-2}. If the minus five-thirds law should be correct, Eq. (6) can be modified following a suggestion by Inoue (1953) to

$$S_w(x)\,dx = \frac{10}{9}\,x\,(1+x)^{-8/3}\,dx \qquad . \quad . \quad . \quad (7)$$

Eq. (7) characterizes a spectrum of greater dispersion than Eq. (6). A final decision as to which relation is preferable depends largely on additional observations at the high-frequency end of the spectrum. Neither equation will fit all spectra perfectly over all frequencies. Both give the correct region for the important part of the spectrum.

Originally, Eq. (6) was not intended to be useful under stable conditions, because then the ground does not determine the vertical extent of the eddies to a height as great as 91 m. Nevertheless, it was discovered that Eq. (6) provided a satisfactory fit in the two stable cases, provided that γ was permitted to vary with the lapse rate or short-wave radiation. However, γ can no longer be expected to be invariant with height. Fig. 17 shows the variation of the parameter γ as function of radiation. γ was determined by fitting Eq. (6) to the observed spectra by eye.

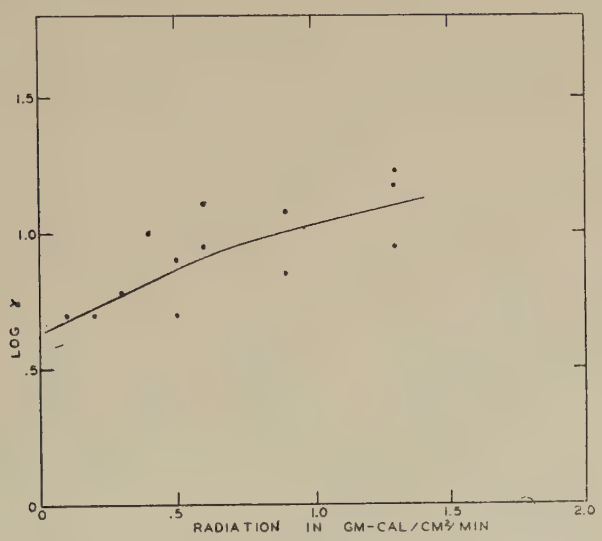

Figure 17. Variation of parameter γ at 91 m with radiation.

Acknowledgments

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Radiation and lag errors of the Meteorological Office radiosonde and the diurnal variation of upper-air temperature

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SUMMARY

Radiation errors of the Meteorological Office radiosonde have been obtained from calculations of the rates of absorption of radiation and of heat loss by forced convection. The effect of the heating of the radiation shield on the temperature of the air reaching the thermometer element becomes very large at high levels. The errors due to direct solar radiation and to radiation reflected and emitted from below an ascending sonde have been calculated for different solar altitudes and for heights up to 30 km.

At solar altitudes between 40° and 60° the errors amount to about 1°C, 2·5°C and 9°C at 5, 15 and 25 km respectively. Observed differences in readings from ascending and rapidly descending sondes indicated that the error at 25 km was within about 7 per cent of the calculated value.

Lag coefficients have been derived from the heat-transfer coefficients and the calculated value for ground level is in good agreement with laboratory measurements.

The errors of the radiosonde can account for the whole of the observed differences between day and night observations up to 15 km. Above that height, however, the differences are appreciably greater than the errors and suggest that the daily range of temperature variation over the British Isles in spring and autumn is about 1·5°C at 20 km and about 3°C at 25 km.

1. Introduction

In his account of the Meteorological Office (Kew pattern) radiosonde Dymond (1947) discussed the accuracy of the temperature measurements and came to the conclusion that the chief systematic errors arise from the effect of solar radiation. Comparison of temperatures at the same height during neighbouring soundings at midday and midnight in summer show differences of about 3°C at 15 km rising to about 6°C at 20 km, the midday readings being the higher. These differences may be due partly to the radiation error of the instrument and partly to any diurnal variation in the temperature of the air itself, and in order to obtain reliable information about the latter it is necessary to know what the radiation error amounts to. Dymond gave some evidence for believing that the instrumental radiation errors are less than 20 per cent of the differences quoted above, but suggested that at greater heights a larger contribution cannot be excluded. Since ascents up to 30 km have become more frequent it is now desirable to obtain more precise information about the radiation errors.

2. Factors affecting radiation errors

The possible effects of radiation on the temperature unit of the Meteorological Office radiosonde (Mark II) can be more readily understood by reference to the drawing of the element and shield reproduced in Fig. 1 (a). The temperature element itself is made of polished steel/invar bimetal, 0·25 mm thick, rolled into the shape of a hollow open-ended cylinder. In the present form of the shield, which is made of thin polished aluminium, the lower half round the element is double-walled but the inner wall is extended upwards by a rectangular duct. This arrangement was adopted after many variations had been tried out by Dymond (1947). Like all radiation shields it is necessarily a compromise, as the following considerations will show.

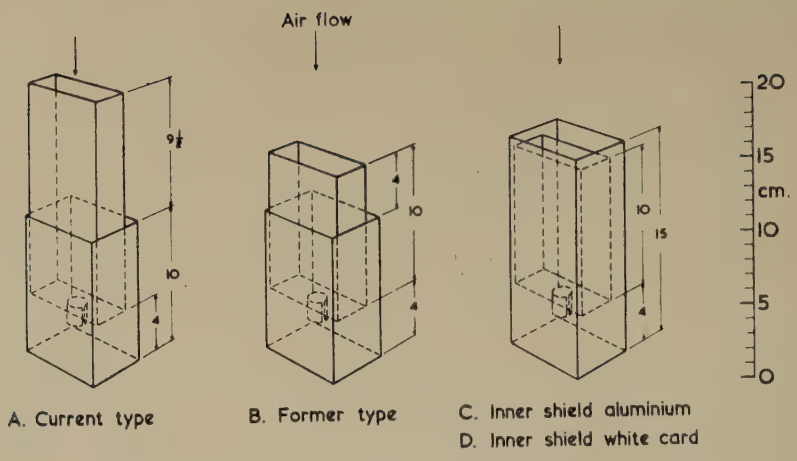

Figure 1. Radiosonde radiation shields.

(*a*) At high solar elevations there is a possibility of solar radiation falling directly on to the element if the top opening of the shield is such as to allow a free flow of air.

(*b*) For the same reason radiation may reach the element after multiple reflection inside the shield.

(*c*) The shield itself may absorb sufficient radiation to warm appreciably the air flowing through it (the effect of heat re-radiated from the shield to the element can be shown to be negligible).

(*d*) There is also the possibility of radiation reflected or emitted from the earth's surface or from clouds being strong enough to heat the temperature unit appreciably.

It is clear that the form of shield which is the more effective in preventing (*a*) and (*b*) is likely to be less effective against (*c*), and vice versa.

Although laboratory experiments to measure the effects of radiation under ground level conditions can be devised it is more difficult to extend such experiments to the conditions to which the radiosonde is subjected at high levels. However, the form of the element and shield adopted for the Meteorological Office radiosonde is such as to enable the effects to be calculated by using heat-transfer theory.

3. Application of theory of heat transfer

The extent to which the temperature T of a thermometer element with a heat capacity C differs from that of the air reaching it is governed by the heat-transfer equation comprising the following terms

$$C\frac{dT}{dt} = Q + H + K \quad . \quad . \quad . \quad . \quad . \quad (1)$$

CdT/dt being the net rate of transfer of heat and the other terms representing the rates of transfer, to or from the element, due to radiation, convection and conduction respectively. Since the supports of the radiosonde bimetal-element are of very small cross-section the effect of conduction is relatively small and we can neglect K without appreciably affecting the final result. This has been confirmed by the fact that insulating the supports of the thermometer element from the frame of the unit made no appreciable difference to its rate of response.

The term Q is the sum of the effects due to direct solar radiation, to radiation reflected from surfaces below the sonde and to long-wave radiation emitted from below. The term H is the product of the heat-transfer coefficient q and the temperature difference between the absorbing body and the air $(T - T_a)$.

Eq. (1) can now be re-written in terms of this temperature difference

$$T - T_a = \frac{Q}{q} - \frac{C}{q}\frac{dT}{dt} \quad . \quad . \quad . \quad . \quad (2)$$

The term Q/q is the temperature rise due to radiation, which is thus obtainable if we know the rate of absorption of radiation and the heat-transfer coefficient. The last term in Eq. (2) is the lag error due to thermal inertia of the body, the ratio C/q being the lag coefficient.

In calculating the radiosonde errors it is necessary to distinguish between (*a*) the heating of the bimetal element itself and (*b*) the heating of the radiation shield and its effect on the air flowing through it. It is only the heating of the inner shield, with its extension beyond the outer shield, which is important, and, in what follows, any reference to the shield should be taken to mean the inner shield unless otherwise stated. The two following sections explain how the heat-transfer coefficients and the rates of absorption of radiation are derived.

4. Heat transfer by forced convection

The heat-transfer coefficient of a body in a moving fluid at a different temperature is given by

$$q = Nu\, k\, S/D \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where Nu is the Nusselt number, k the thermal conductivity of the fluid, S the area exposed to convective heat transfer and D the characteristic dimension of the body. The Nusselt number is the ratio of the rate of convective heat transfer to the rate of transfer by conduction through the fluid and the values of Nu appropriate to the form and dimensions of the body and the mass rate of fluid flow can be calculated from empirical formulae given in text-books such as that of Fishenden and Saunders (1950). The formulae given in Table 1 have been used in the present investigation. Re is Reynolds's number given by $v\rho D/\mu$, where v is the rate of air flow, ρ the air density and μ its dynamic viscosity. The numerical constants in these formulae are dependent on Prandtl's number (given by $C_p\,\mu/k$, where C_p is the specific heat of the fluid at constant pressure) which has the value of 0·71 for air over the range of temperature and pressure in atmospheric conditions up to 30 km.

TABLE 1. Formulae for Nusselt numbers

Form of surface	Characteristic dimension	Re	Nu
Flat plate	Length in direction of flow	$< 2 \times 10^4$	$0{\cdot}59\, Re^{0{\cdot}5}$
		$> 2 \times 10^4$	$0{\cdot}032\, Re^{0{\cdot}8}$
Duct, with effective diameter d and length l	Effective diameter	$< 2{\cdot}1 \times 10^3$	$1{\cdot}45\,(Re\, d/l)^{0{\cdot}33}$
		$> 2{\cdot}1 \times 10^3$	$0{\cdot}020\, Re^{0{\cdot}8}$
Cylinder, perpendicular to air flow	Diameter	10^{-1} to 10^3	$0{\cdot}32 + 0{\cdot}43\, Re^{0{\cdot}52}$
		10^3 to 5×10^4	$0{\cdot}23\, Re^{0{\cdot}6}$

The flat-plate formulae can be applied to obtain the heat-transfer coefficient q_e for the inner and outer surfaces of the bimetal element in the shield, the characteristic dimension D being the length in the direction of the air stream through the shield. For the coefficient q_i for the inner surface of the shield the duct formulae are applicable, with the effective diameter as D. The flat-plate formulae would be appropriate for the outer surface of the shield if the sonde ascended vertically. Generally, however, it swings at the end of its 40 m suspension, with an amplitude of about $\pm 30°$, so there is a component of air flow perpendicular to the axis of the shield varying between 0 and 10 m/sec. The average resultant air flow is about 9·5 m/sec at an angle of about 45° to the axis. There is little difference between the heat-transfer coefficient q_p, given by the flat-plate formula, for an air flow parallel to the axis and the coefficient q_c, given by the cylinder formula, for the same rate of air flow perpendicular to the axis. The mean of q_p and q_c for a rate of flow of 9·5 m/sec has therefore been taken as representing the coefficient q_0 for the outer surface of the shield.

Table 2 gives the dimensions used in obtaining the heat-transfer coefficients.

TABLE 2. DIMENSIONS USED IN CALCULATING HEAT TRANSFER COEFFICIENTS

Surface	Aerodynamic shape	Effective length (cm)	Width (perimeter) (cm)	Effective diameter (cm)	Surface area (cm^2)
Bimetal element, inner and outer surfaces	Flat plate	1·6	3·0	–	9·6
Shield, inner surface	Duct	15	–	3·3*	157
Shield, outer surface, flow parallel to axis	Flat plate	10	15	–	150
Shield, outer surface, flow perpendicular to axis	Cylinder	8·5	–	4·7†	127

* Effective diameter of duct of rectangular cross-section = 4 × cross-section/perimeter.

† Effective diameter of rod of rectangular cross-section = perimeter/π.

TABLE 3. HEAT-TRANSFER COEFFICIENTS, q, FOR A SONDE ASCENDING AT 6 M/SEC AND SWINGING WITH AN AMPLITUDE OF $\pm 30°$. (For the q's and for mC_p the unit is cal sec^{-1} $(°C)^{-1}$ × 10^{-4})

Height km	Pressure mb	Density × 10^6 g cm^{-3}	Viscosity × 10^6 g cm^{-1} sec^{-1}	Thermal conductivity × 10^4 cal cm^{-1} sec^{-1} $(°C)^{-1}$	Shield					Air in shield		Element
					q_p	q_c	q_0	q_i	q_s	mC_p	B	q_e
30	12	18	147	49	15·2	15·8	15·5	13·0	25·8	23·8	·420	1·90
25	25	39	146	49	21·9	22·3*	22·1	16·3	36·2	50	·280	2·77
20	55	88	144	48	31·4	34·6*	33·0	20·5	51·7	106	·175	3·95
15	115	184	145	48	46·9	50·8*	48·9	24·1*	71·8	238	·097	5·9
10	264	413	146	49	71	90·7*	81	45·6*	125	520	·084	8·9
5	531	723	163	54	136*	135*	135	75·5*	208	946	·077	12·7
0	1,013	1,225	179	60	201*	184*	192	112*	301	1,530	·071	16·9

* The Reynolds numbers in these cases are above the critical values given in Table 1; in the other cases they are below these values.

q_p refers to flow along outer surface parallel to axis.

q_c refers to flow over outer surface perpendicular to axis.

q_0 is the mean of q_p and q_c.

q_i refers to flow over inner surface.

$q_s = q_0 + mC_pB$ = total transfer coefficient for shield.

mC_p is the mass rate of heat flow in the air through the shield.

$B = (T_a^1 - T_a)/(T_s - T_a)$ i.e. ratio of heating of air in shield to the heating of the shield.

q_e refers to flow over inner and outer surfaces of element.

The values of the coefficients for a sonde ascending at 6 m/sec and swinging through an angle of $\pm$ 30° have been calculated for various heights and are given in Table 3. With the rate of ascent constant the basic variables are the density, viscosity and thermal conductivity of the air; the values of these used are included in Table 3.

As there is relatively little variation of viscosity and thermal conductivity with height, especially above 10 km, it is the air density which is almost wholly responsible for any change in the heat transfer, and therefore the radiation error, with height.

In applying the calculated heat-transfer coefficients in Eq. (2) there is a complication arising from the fact that whereas the formulae for the Nusselt numbers for the flat plate and cylinder are based on temperature differences $(T - T_a)$ between the body and the undisturbed air upstream, the formulae for the duct refer to the mean of the differences at the inlet and outlet. Further, since the air temperature in the duct varies across the cross-section, a mean value T_a^1 corresponding to perfect mixing is assumed. It is this temperature which is taken to be that of the air reaching the bimetal element. Applying, then, Eq. (2) to the case of the shield we have

$$C_s \frac{dT_s}{dt} = Q_s - q_0 (T_s - T_a) - q_i [T_s - (T_a + T_a^1)/2] \quad . \quad (4)$$

The last term, representing the rate of heat loss by the inner surface, must be equal to the rate of gain of heat by the air passing through the shield, which is $mC_p (T_a^1 - T_a)$, where m is the mass rate of air flow, i.e. $\pi D^2 \rho v/4$, D being the effective diameter of the duct. It follows that

$$\frac{T_a^1 - T_a}{T_s - T_a} = \frac{2q_i}{2mC_p + q_i} = B \quad . \quad . \quad . \quad (5)$$

thus giving the ratio of the heating of the air to the heating of the shield. It also follows that

$$q_i [T_s - (T_a + T_a^1)/2] = mC_p B (T_s - T_a) \quad . \quad . \quad (6)$$

We can therefore now express the heat balance of the shield by

$$C_s \frac{dT_s}{dt} = Q_s - q_s (T_s - T_a) \quad . \quad . \quad . \quad (7)$$

where q_s, the total heat-transfer coefficient, is given by

$$q_s = q_0 + mC_pB$$

The calculated values of q_0, mC_p, B and q_s are included in Table 3. An experimental determination of the value of B was attempted by measuring the temperature of air passing through a heated shield in a wind tunnel. The shield was maintained about 5°C warmer than the air stream, the speed of which was 6 m/sec. Measurements were made with the axis of the shield not only in the direction of the air flow but also displaced 30° in horizontal and vertical planes. Temperatures were measured by thermocouples but as the experiments could only readily be made at ground-level pressure the temperature differences between the air entering and leaving the shield were very small and there was a large scatter in the individual readings. The mean, however, of 24 measurements gave the value of B as 0·08 $\pm$ 0·08 (P.E.) which is not inconsistent with the calculated value of 0·071.

5. Rate of absorption of radiation

The radiation term Q in the heat-balance equation depends, of course, on the intensity of the incident radiation I, on the absorption factor A of the body and on the orthogonal projection of the area exposed to the beam. The computation of Q is complicated by

(*a*) the swinging of the sonde, (*b*) shading of the shield or element by other parts of the sonde, and (*c*) multiple reflections of radiation entering the open ends of the shield.

For the absorption of solar radiation of intensity I by the element or the shield we can write

$$Q_h = AIf(S_d, h, A_s)\cos h \quad . \quad . \quad . \quad . \quad (8)$$

where h is the solar altitude, S_d the effective vertical area exposed, including that receiving radiation reflected internally, and A_s is the absorption factor of the shield.

The numerical values of $f(S_d, h, A_s)$ are best obtained by geometrical measurement.

The rate of absorption of radiation reflected from the ground or from cloud is given by

$$Q_m = AI_r \int_{\alpha_1}^{\alpha_2} \int_{\phi_1}^{\phi_2} \cos^2\phi \, d\phi \, S_m \, d\alpha \quad . \quad . \quad . \quad (9)$$

where S_m is the effective vertical area of the shield or element exposed, which may vary with the azimuth and elevation angles, α and ϕ, of the effective portions of the radiating surfaces as 'seen' by the sonde. I_r is the intensity of the reflected radiation and is given by

$$I_r = RI \sin h \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where R is the albedo of the reflecting surface and I is the intensity of solar radiation at the surface.

The rate of absorption of radiation Q_l emitted from below is also given by (9) except that the intensity of the long-wave radiation I_l and the appropriate absorption factor A_l must be used. The intensity I_l is given by

$$I_l = \sigma(T_0^4 - T^4)/\pi \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where σ is the Stefan-Boltzmann coefficient ($1{\cdot}37 \times 10^{-12}$ cal cm^{-2} sec^{-1} °C^{-4}) and T_0 and T are the temperatures of the emitting surface (cloud or ground) and the sonde. The element and shield themselves emit a negligible net amount of radiation to the surrounding atmosphere.

The angular limits α_1, α_2, ϕ_1, ϕ_2, in (9) are governed by the extent of the shading of the element and the shield by other parts of the sonde. For evaluating Q_m and Q_l the integrations with respect to $d\phi$ have been done in 10° steps, and the effect of swinging of the sonde has been taken into account. The intensity of solar radiation is taken to be 0·032 cal cm^{-2} sec^{-1} at the top of the atmosphere but in the final results it is adjusted for air mass appropriate to the solar altitude and the height of the sonde by means of a graph provided by Väisälä (1948). In evaluating I_r a value of 0·35 for R has been taken as representing average conditions of the earth's surface and cloud cover for the British Isles. The long-wave radiation I_l has been calculated for a surface temperature T_0 of 288°K and a sonde temperature T of 220°K; these conditions cover most soundings in the British Isles except at heights below 10 km.

The absorption factors of the temperature element and the shield were measured by the National Physical Laboratory at various wavelengths between 0·3 and 10 μ for both normal and 45° incidence, a random sample of each material being taken from stock. From these measurements and from the spectral distribution of solar energy the absorption factors were found to be 0·33 for the aluminium shield and 0·75 and 0·60 for the two sides of the bimetal element. A mean of 0·67 has been used for the element. The values for 45° incidence did not differ from those at normal incidence by more than 0·02. The N.P.L. values for a wavelength of 10 μ, viz. 0·03 for the shield and the mean of 0·18 for the element have been used for the absorption factors for the long waves. It must be borne in mind that there may be appreciable differences between individual samples,

depending on the state of their surfaces. For a white card shield used in some experimental soundings to be described later, factors, based on published data, of 0·25 for solar radiation and 0·95 for long waves have been used.

Calculated rates of absorption of radiation by the shield and by the element inside it for different solar altitudes are given in Table 4.

TABLE 4. RATE OF ABSORPTION OF RADIATION IN CAL $SEC^{-1} \times 10^{-3}$

Solar altitude ($h°$)	10°	30°	50°	70°	90°
Shield					
Direct solar (Q_h)	543	597	543	422	415
Reflected from below (Q_m)	32	93	141	174	185
Emitted from below (Q_l)	8	8	8	8	8
Total (Q_s)	583	698	692	604	608
Element					
Direct solar (Q_h)	0.9	4·1	7·8	9·4	12·2
Reflected from below (Q_m)	0·7	2·0	3·0	3·7	4·0
Emitted from below (Q_l)	0·5	0·5	0·5	0·5	0·5
Total (Q_e)	2·1	6·6	11·3	13·6	16·7

The figures for the absorption of reflected radiation are for an albedo 0·35 and those for long-wave radiation refer to a surface at 288°K with the sonde at 220°K.

The values for direct radiation take account of the swinging of the sonde through an angle of 30°. This swinging makes no appreciable difference to the amount of radiation reaching the sonde from below. The values are plotted in Fig. 2 for comparison with values relating to a sonde remaining vertical. The curves are similar in shape to those given by Väisälä (1941) for his design of sonde.

The effect of radiation emitted from below is quite negligible in the case of the shield, but the contribution by the reflected radiation to the total absorption increases from near zero at low solar altitudes to 30 per cent at 90°. In the case of the element the very small amount of radiation absorbed at low solar altitudes is roughly half due to the direct radiation entering the upper opening of the shield and half to radiation reflected or emitted from below. But at high solar altitudes the direct radiation contributes about three-quarters and reflected radiation one quarter to the total, the effect of the emitted radiation then being less than five per cent. It must be remembered that at low solar altitudes the intensity of the sun's radiation is reduced appreciably below the value 0·032 cal cm^{-2} sec^{-1} assumed in Table 4. At solar altitudes above 20° and heights about 10 km the reduction is less than 10 per cent.

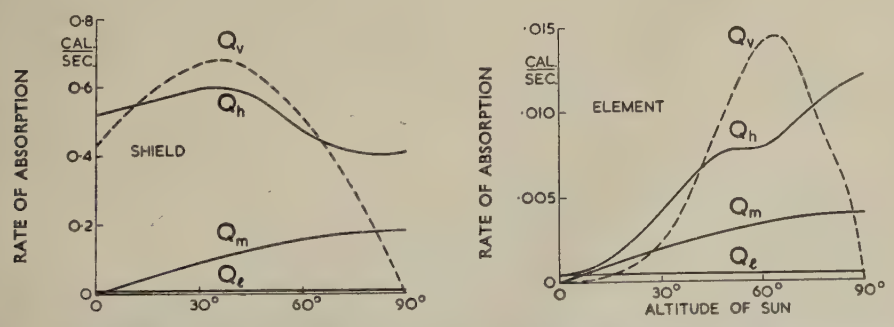

Figure 2. Radiation absorbed by element and shield. Q_v – direct solar radiation; sonde vertical. Q_h – direct solar radiation; sonde swinging ± 30°. Q_m – reflected solar radiation; albedo 0·35 (other values in proportion). Q_e – long-wave radiation; earth at 288°K, sonde at 220°K (effect of swinging on Q_m and Q_e is negligible).

6. Calculated radiation errors

The values of q_s, q_e, B, Q_s and Q_e given in Tables 3 and 4 enable us to obtain the radiation terms Q_s/q_s and Q_e/q_e in the equations

$$T_s - T_a = \frac{Q_s}{q_s} - \frac{C_s}{q_s}\frac{dT_s}{dt} \text{ and } T_e - T_a^1 = \frac{Q_e}{q_e} - \frac{C_e}{q_e}\frac{dT_e}{dt} \quad . \quad (12)$$

But whereas Q_s/q_s gives the radiational heating of the shield above the free-air temperature, Q_e/q_e only gives the heating of the element above the temperature T_a^1 of the air in the shield. The definition of B in Eq. (5), however, enables us to eliminate T_a^1 and obtain the heating of the element above the free-air temperature, thus :

$$T_e - T_a = \frac{Q_e}{q_e} + \frac{BQ_s}{q_s} - \frac{C_e}{q_e}\frac{dT_e}{dt} - \frac{BC_s}{q_s}\frac{dT_s}{dt} \quad . \quad . \quad . \quad (13)$$

The terms $(Q_e/q_e + BQ_s/q_s)$ together represent the total radiation error of the bimetal element in the shield. The calculated values of this error for a selection of heights and solar altitudes are given in Table 5, which also includes values for the heating of the shield and the air in it.

TABLE 5. Calculated radiation errors of the Meteorological Office radiosonde. Combining the effects of direct solar radiation, reflected radiation from earth and/or cloud with average albedo 0·35, and long-wave radiation emitted from the earth and/or cloud.

Solar altitudes	Height km	10°	30°	50°	70°	90°
				°C		
(*a*) Heating of shield, Q_s/q_s	30	22·6	27·1	26·9	23·4	23·6
	20	11·3	13·5	13·4	11·7	11·8
	10	4·7	5·6	5·6	4·8	4·9
	0	1·9	2·3	2·3	2·0	2·0
(*b*) Heating of air in shield, BQ_s/q_s	30	9·6	11·4	11·3	9·9	10·0
	20	2·0	2·4	2·3	2·0	2·1
	10	0·4	0·5	0·5	0·4	0·4
	0	0·1	0·2	0·2	0·1	0·1
(*c*) Error of element with respect to air in shield, Q_e/q_e	30	1·1	3·5	6·0	7·2	8·8
	20	0·5	1·7	2·9	3·4	4·2
	10	0·2	0·7	1·3	1·5	1·9
	0	0·1	0·4	0·7	0·8	1·0
(*d*) Error of element with respect to free air, $Q_e/q_e + BQ_s/q_s$	30	10·6	14·9	17·3	17·1	18·8
	25	5·2	7·7	9·3	9·5	10·7
	20	2·4	4·0	5·1	5·3	6·2
	15	1·1	1·9	2·7	3·0	3·5
	10	0·5	1·1	1·7	1·8	2·2
	5	0·3	0·7	1·1	1·2	1·4
	0	0·1	0·4	0·7	0·7	0·6

In (*a*), (*b*) and (*c*) the intensity of solar radiation is taken to be 0·032 cal cm^{-2} sec^{-1}. In (*d*) account is taken of the effect of air mass on the intensity.

It will be seen that for solar altitudes about 50° the heating of the air in the shield is responsible for about 20 per cent of the radiation error near the ground, 30 per cent at 10 km and 65 per cent at 30 km. The contributions are higher still at lower solar altitudes since the angle of incidence of solar radiation on the shield is nearer to normal incidence. The remainder of the error is, of course, due to radiation falling on the element itself, either directly or after reflection by the inner surface of the shield.

The albedo of 0·35 adopted for Table 5 is a rough average value for earth surfaces about half obscured by cloud. Over the sea the albedo may be as low as 0·1 and over

snow or thick cloud as high as 0·6, if not higher. Calculation of the radiation errors corresponding with these limits shows that a change of 0·1 in the albedo alters the errors by about 7 per cent, except at solar altitudes less than about 20°, when the effect becomes much smaller.

7. Lag errors

The lag in temperature of the shield and the element is represented by the last terms in Eqs. (12), C/q being the lag coefficient which is usually written as λ. Since C is readily obtained from the mass of the body and its specific heat we can obtain λ from the calculated value of q. The lag coefficient can also be measured in the laboratory (where radiational exchange can be made negligible) by observing the time required for an initial temperature difference between the thermometer element (or shield) and the air to be reduced in the ratio $1/e$. In such conditions

$$T - T_a = (T_0 - T_a) \exp\left(-\frac{t}{\lambda}\right) \quad . \quad . \quad . \quad (14)$$

where $T = T_0$ at $t = 0$. This measurement of λ therefore affords a check on the calculated heat transfer coefficient.

For the bimetal element without its shield the measured value of λ_e in an air flow of 6 m/sec at ground level pressure was 7·3 sec. Under the same conditions the calculated value of q_e of $16{\cdot}9 \times 10^{-3}$ (from Table 3), with $C = 0{\cdot}116$, gives $\lambda_e = 6{\cdot}9$ sec. The agreement is reasonable.

When the thermometer element is in its shield, however, its lag behind the free-air temperature will be affected by the lag λ_s of the shield itself and by the extent to which the shield heats the air reaching the element. The total lag error is, in fact, given by the last two terms of Eq. (13), but it is not readily evaluated since dT_s/dt is not normally measured. The lag of the element in the shield can, however, be measured in the laboratory. From Eq. (14) and Eqs. (12) for the shield, together with the definition of B, it can be shown that for a sudden temperature difference in the absence of radiational exchange

$$\frac{dT_e}{dt} = \frac{1}{\lambda_e}\left[T_e - T_a - B\,(T_0 - T_a) \exp\left(\frac{-t}{\lambda_s}\right)\right] \quad . \quad . \quad (15)$$

and that

$$T_e - T_a = (T_0 - T_a)\left(\exp\frac{-t}{\lambda_e}\right)\left[1 + \frac{B\lambda_s}{\lambda_s - \lambda_e}\left\{\exp\left(\frac{t}{\lambda_e} - \frac{t}{\lambda_s}\right) - 1\right\}\right] \quad (16)$$

Now when λ_s is not very different from λ_e, which is the case with the radiosonde unit, and when the measurements are made with t not greatly exceeding λ_e, this simplifies to

$$T_e - T_a = (T_0 - T_a) \exp\left[-\frac{t\,(1 - B)}{\lambda_e}\right] \quad . \quad . \quad . \quad (17)$$

In other words the lag coefficient of the shielded element λ_{es} is $1/(1 - B)$ times that of the unshielded element and the total lag error can now be written

$$\lambda_{es}\frac{dT_e}{dt} \quad \text{or} \quad \frac{\lambda_e}{1 - B}\frac{dT_e}{dt}$$

The laboratory measurement of λ_{es} under the same conditions as for λ_e gave a value of 7·6 sec. The calculated value given by $C_e/q_e\,(1 - B)$ is 7·4 sec.

It is difficult to measure the lag coefficient in laboratory conditions corresponding with the normal rate of ascent at very low air densities, but it is reasonable to expect that the method of calculation which has been found to give satisfactory agreement at

ground level should also be satisfactory for high-level conditions. Table 6 gives the lag coefficients calculated for various heights and the normal rate of ascent of 6 m/sec. The values of the heat capacities used were 0·116 at 288°K and 0·106 at 220°K for the element and 2·42 and 2·16, at these temperatures, for the shield. The table also includes the lag errors for typical rates of change of indicated temperature in night and day soundings. In the troposphere the errors are the same sign as, and about one-third the magnitude of, the radiation errors. In the stratosphere they are very small in comparison with the radiation errors and are of opposite sign.

TABLE 6. LAG COEFFICIENTS AND ERRORS (RATE OF ASCENT 6 M/SEC)

Height	Coefficients (sec) λ_e	λ_s	λ_{es}	Typical errors (°C) night	day
30	56	84	97	− 0·6	− 1·6
25	38	60	54	− 0·3	− 0·7
20	27	42	33	− 0·2	− 0·3
15	18	30	20		
10	11·9	17·3	12·9	+ 0·5	+ 0·5
5	8·7	11·0	9·5	+ 0·3	+ 0·3
0	6·9	8·0	7·4	+ 0·3	+ 0·3

The lag errors are for a temperature gradient of − 6°C/km below 15 km and + 1°C/km above 15 km. The daytime errors take account of the rate of change of temperature of the element by radiational heating for solar altitude 40°.

8. OBSERVED ERRORS AT HIGH LEVELS

Some comparisons of temperatures indicated at the same height by a radiosonde ascending at the normal rate and descending at a much higher rate have afforded some confirmation of the existence of the large radiation errors at high levels. Such comparisons were made in five soundings at Downham Market (52°36′N, 0°24′E) in September, 1950 when the balloons burst at heights near 30 km and the parachutes failed to open immediately. The sondes were tracked by radar. Fig. 3 is a graph of the readings of temperature and height obtained during one of the soundings and shows the sudden fall in the temperature readings during the rapid descent.

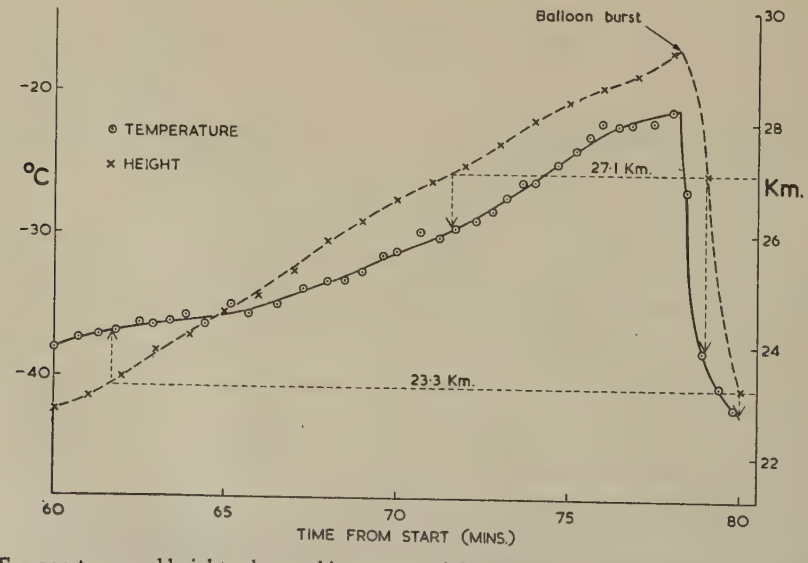

Figure 3. Temperatures and heights observed in ascent and descent at Downham Market, 10 hr 21 Sept. 1950.

In the descent the heating, by the shield, of the air reaching the element is negligible since the latter is near the bottom opening of the shield. The temperature T_d indicated by the element during the descent is therefore given by

$$T_d - T_a = \frac{Q_e}{q_d} - \frac{C_e}{q_d}\frac{dT_d}{dt} \quad . \quad . \quad . \quad . \quad (18)$$

where q_d is now the much larger heat-transfer coefficient of the element for the much higher rate of air flow V.

Combining Eqs. (13) and (18) then gives the difference between the ascent and descent temperatures

$$T_e - T_d = \frac{Q_e}{q_e} + \frac{BQ_s}{q_s} - \lambda_{es}\frac{dT_e}{dt} - \frac{Q_e}{q_d} + \lambda_d\frac{dT_d}{dt} \quad . \quad . \quad (19)$$

This can be evaluated from the calculated values of the radiation errors and the lag coefficients and the observed rates of change of indicated temperatures. The results of the calculations and comparisons are given in Table 7. It should be remembered that the observations for each height are single pairs of readings and considerable scatter in the results must be expected. The calculation of the lag error in the descent is probably the most doubtful factor in the computations, since the rapid rate of change of temperature is difficult to measure accurately. However, the mean difference of 0·6°C between the observed and calculated temperature differences can be considered reasonably good and

TABLE 7. OBSERVED AND CALCULATED ERRORS (SOLAR ALTITUDE 40°)

Date 1950 Sept.	Height (km)	Rate of descent m/sec	Calculated errors in °C: Ascent Radiation	Ascent Lag	Ascent (T_e-T_a)	Descent Radiation	Descent Lag	Descent (T_d-T_a)	Ascent – descent (T_e-T_d): Calc.	Obs.	O–C
12	24·0	41	7·9	0·2	7·7	1·2	− 0·7	1·9	5·8	6·1	+ 0·3
12	21·8	34	6·1	0·3	5·8	1·1	− 0·2	1·3	4·5	5·2	+ 0·7
13	28·2	57	13·4	2·8	10·6	1·4	− 1·3	2·7	7·9	8·8	+ 0·9
13	25·8	47	9·8	1·3	8·5	1·3	− 0·6	1·9	6·6	6·0	− 0·6
19	28·8	61	14·5	2·0	12·5	1·4	− 2·9	4·3	8·2	10·3	+ 2·1
19	25·2	44	9·1	1·3	7·8	1·3	− 1·1	2·4	5·4	5·7	+ 0·3
20	27·8	55	12·7	1·3	11·4	1·4	− 2·1	3·5	7·9	9·4	+ 1·5
20	24·3	42	8·2	0·5	7·7	1·2	− 0·7	1·9	5·8	4·5	− 1·3
21	27·1	52	11·6	1·1	10·5	1·3	− 1·4	2·7	7·8	8·9	+ 1·1
21	23·3	38	7·2	0·4	6·8	1·2	− 0·2	1·4	5·4	6·0	+ 0·6
Mean	25·6	47	10·0	1·1	8·9	1·3	− 1·1	2·4	6·5	7·1	+ 0·6

it might well be accounted for by the albedo being different from the value 0·35 assumed in the calculations. The soundings were made with the sky about half clouded but the surface exposed to the sondes through the breaks in the cloud was part grassland and part sea.

9. COMPARISONS WITH MODIFIED SHIELDS

In order to see whether radiation errors could be appreciably reduced by modifying the design of the shield a series of comparisons of two sondes, one standard and one modified, on the same balloons was carried out at Downham Market in Sept. 1950. The modifications used are shown in Fig. 1. The series also included sondes without any shield, i.e., with the element fully exposed except for partial shading by the sonde. Five or six soundings to high levels were made with each modification. Calculated radiation errors for the standard and for the modified sondes at solar altitudes 40° are plotted in Fig. 4. The results of the comparisons are given in Fig. 5 in which the points on the graphs represent the means of about 5 observations of the differences (except at the

highest levels where fewer were obtained). Probable errors of the mean differences are also indicated. The calculated differences are derived from Fig. 4 and do not include the lag errors, which, however, would not affect the observed differences by more than 0·3°C. The agreement between the observations and the calculations is generally within 1°C. The differences that the modifications make, especially below 20 km, are relatively small. Dymond (1947) found much the same result in some earlier experiments with modified shields. It emphasizes the fact, already mentioned, that the design has to be a compromise between conflicting requirements. Thus, reducing the length of the shield, as in modification B, decreases the heating effect of the shield but exposes the element more to direct radiation. Further decrease of the radiation falling on the shield, by extending the outer shield as in modification C, appears to be nullified by decrease in the heat convected from the outer surface of the inner shield. Moreover, neither B nor

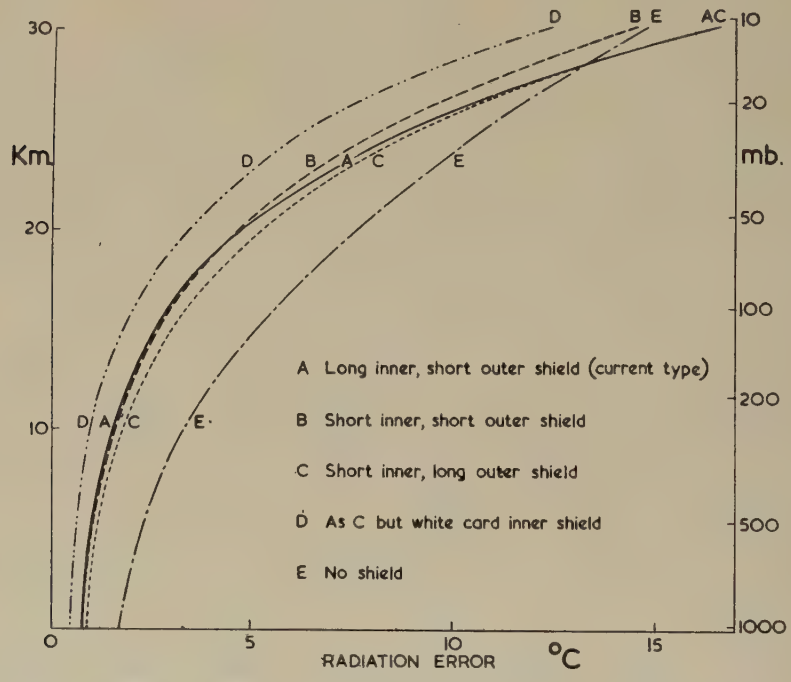

Figure 4. Calculated radiation errors with standard and modified shields (solar altitude 40°).

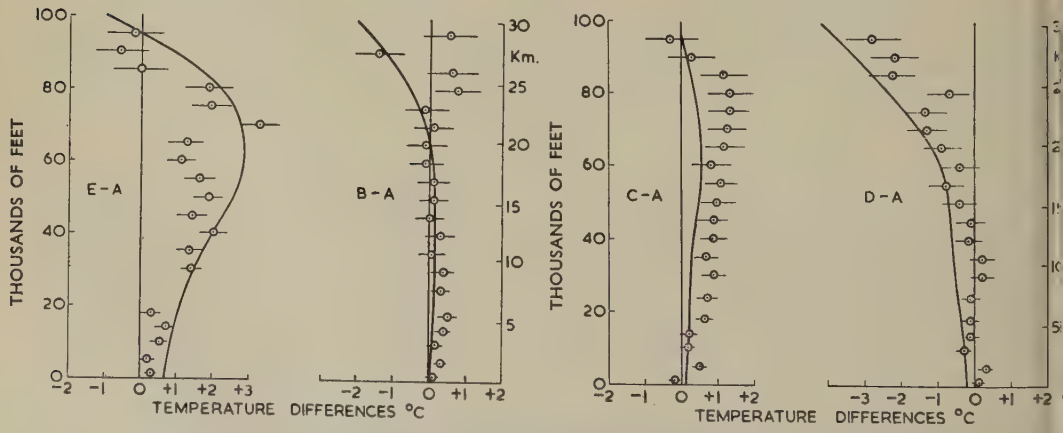

Figure 5. Observed and calculated temperature differences with standard and modified shields (solar altitude 40°).

C can make any difference to the amount of radiation absorbed inside the shield. The effect of the heating of the standard pattern shield increases so rapidly at high levels that at 30 km the radiation error is as large as when the element is fully exposed. The only modification that showed any appreciable improvement – and this only at levels above 20 km – was D, in which the inner shield was made of white card instead of aluminium. This is probably due to the card having a lower absorption factor for solar radiation and to its being a diffuse reflector and so reflecting less radiation on to the element.

10. The diurnal variation of upper-air temperature

If the radiation and lag errors are correctly evaluated the magnitude of the diurnal range of air-temperature variation, and its change with height, should be obtainable from the differences between day and night radiosonde observations. In Fig. 6 the curve gives the combined radiation and lag errors for a solar altitude of 40°. In the same graph are plotted four series of mean differences between night and day observations, the latter being made at about this solar altitude. With the exception of those derived from three pairs of high altitude soundings at Downham Market on 15, 25 and 29 September, 1950, the means are for routine soundings for the six months February to April and August to October combined. Those published by Kay (1951) represent about 1,800 observations in 1947 and 1948 at five stations in the British Isles. Piegza's (1947) means are derived from about 700 observations in 1943-6 at Fazakerley (53°28′N, 2°55′W). The means for the routine soundings at Downham Market represent about 1,400 observations in the period 1943-51. It is clear that up to about 15 km radiation errors could account for the whole of the day and night differences. Above that height the comparisons indicate that the differences are too big to be accounted for wholly by the errors, if these are correctly assessed, and that there is a diurnal range of about 1·5°C at 20 km (55 mb) increasing to about 3°C at 25 km (25 mb). We cannot, of course, regard the day and night differences above 22 km as representative of average conditions since they were based on only three comparisons and although each was made on the same day, at 10 hr and 22 hr, they may well be affected by day-to-day temperature

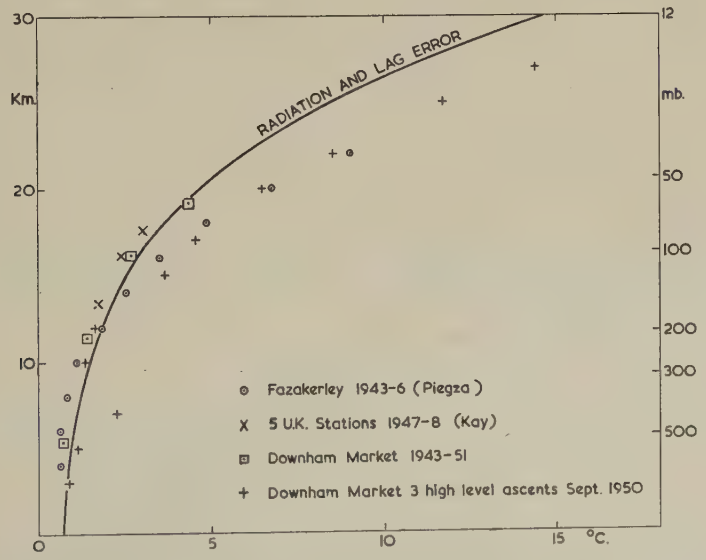

Figure 6. Day- and night-temperature differences compared with instrument errors (solar altitude about 40°).

changes. On the other hand much the same differences above 22 km were obtained in the means of 15 day and 15 night soundings in September and October which were not made on the same days.

The diurnal ranges at 20 km and 25 km indicated by these results are ten times as large as those recently calculated by F. S. Johnson (1953) from the heating due to absorption of solar energy by atmospheric ozone. He used direct measurements of ozone concentration and ultra-violet spectral intensity of sunlight during a rocket flight over New Mexico in June 1949 and, assuming a constant rate of energy loss, found the diurnal temperature change to have a maximum value of 5·3°C at 48 km falling to 1°C at 31 km and 70 km.

Goldie (1951) has suggested the possibility of the diurnal temperature change being due to dynamical causes, in which case the diurnal range should vary approximately inversely with the pressure (or density) and should, therefore, increase exponentially with height. An adiabatic heating of 1·5°C at 20 km would require a pressure increase of 1·3 mb at that level and this same increase would also account for the heating of 3°C at 25 km. An inflow of 1 or 2 mb of air above these levels, with a corresponding outflow at low levels, is not unreasonable. Clearly, however, further information at high levels is required before any safe conclusions about diurnal variation can be drawn.

11. Conclusion

Little advantage is to be gained by introducing any further modifications to the radiation shield as they cannot reduce the radiation errors to negligible proportions. A better course is to use a temperature element for which the ratio of the rates of absorption of radiation and heat convection is so small that the shield becomes unnecessary. The factors favouring this are (*a*) low absorption factor, (*b*) small effective surface area exposed to radiation, and (*c*) large characteristic dimension for the Nusselt number. In general, (*b*) and (*c*) are mutually antagonistic, but a thin plate which would favour (*c*) would also favour (*b*) if its edge could be kept facing the source of radiation. This, of course, is impracticable in a radiosonde and probably the best compromise is a thin wire element perpendicular to the air flow. Calculations show that the maximum radiation error of such an element of diameter 10^{-3} cm at 30 km would be only 0·35°C even if its absorption factor were unity. An element of this type, using wire of $1{\cdot}35 \times 10^{-3}$ cm diameter, is to be used in the form of a coiled coil in the recently designed radar sonde (Grant 1953) and it is hoped that the radiation errors will be small enough to allow the magnitude of the diurnal variation of temperature to be determined accurately.

Acknowledgments

I am indebted to Dr. D. N. Harrison for helpful advice; to Mrs. J. Seaton for assisting in the computations; and to the staff of the Downham Market radiosonde station who undertook the special soundings. The paper is published by permission of the Director of the Meteorological Office.

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Radar echoes from lightning

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(Manuscript received 19 March 1954, in revised form 1 July 1954)

Summary

Photographic observations of radar echoes received from lightning are given and are used to suggest a possible distribution of the electric charge within a thunderstorm cloud.

1. Introduction

Radar echoes attributable to lightning flashes have been observed occasionally on the 10-cm wavelength height-range equipment at East Hill, but being transient echoes, successful photographs were not obtained. The chances of a sweeping aerial pointing at exactly the right part of a cloud at exactly the time of a lightning flash are, of course, small but on the occasions when the echo has been observed in this way it has shown as an intense echo of small extent, near to but outside the top of the strong column of echo received from the most active part of the cumulonimbus itself. Similar echoes have been reported from Canada (Marshall 1953).

With the aerial stationary the chances of receiving the echo, provided the aerial is inclined to the vertical at the correct angle, are greatly enhanced. Ligda (1950) has described echoes received in this way by training his aerial on the column of maximum echo intensity from the cumulonimbus with the elevation adjusted so that the echo intensity was half the saturation value of the A-scope display used. Browne (1951), with a vertically directed radar, observed an echo attributed to lightning at 12,000 ft during a violent thunderstorm. Other observations have been reported from S. Africa (Miles 1953, Hewitt 1953). From time to time attempts have been made at East Hill to obtain such echoes and on 12 August 1953 success was achieved by pointing the aerial nearly at the top of a cumulonimbus echoing column and changing the azimuth of the aerial slightly from that giving the echo of maximum intensity. The echoes were displayed on an A-scope (which indicates amplitude of received signal against range) and showed as transient echoes of high intensity. The fact that the echoes could be attributed to lightning was conclusively proved by listening on R/T and noting that a characteristic lightning atmospheric coincided with the appearance of the transient echo. That the received signal is really an ' echo ' and not a direct radiation at 10-cm wavelength from the lightning flash is proved by the fact that all the signals received were associated with ranges identical with those from which normal weather echoes were being received. Had the signals come from direct radiations on 10-cm wavelength they would have been received from apparently random ranges and independently of the transmitted radar pulse which triggers the time-base of the A-scope. An echo might arise by scattering from the strong gradient of refractive index set up at the surface of the volume of air intensely heated by the lightning flash. Browne (1951) however considers that the signal received in this way would be far too small to account for the observed signal intensities and attributes the strong signal received to scattering from the electrons forming the ionised path of the lightning flash, an interpretation supported by Miles (1953) and Hewitt (1953).

Having once succeeded in obtaining lightning echoes, it was easy to repeat the experiment during thunderstorms on 20 and 21 August, particularly good photographs of the transient echoes being obtained on the last day.

2. Echoes on 12 and 20 August

Lightning echoes on 12 August were received with the aerial pointing at the anvil near to a cumulonimbus echo column. The heights of the centre of the beam at the ranges at which the lightning echoes occurred were 31,000 ft at a range of 65 mi and 35,000 ft at a range of 81 mi. Beam widths, measured to half power, at these ranges would be about 9,000 ft and 11,000 ft respectively. The echo at 81 mi therefore almost certainly originated above 30,000 ft where the temperature was about — 36°C.

On 20 August the echoes from the cumulonimbus clouds were much smaller than on the other two days, showing on the PPI as a narrow line of clearly defined individual cells. Lightning echoes were observed at ranges from 50 to 91 mi and at heights of the centre of the beam from 15,000 ft to 29,000 ft. On this occasion the lightning echo was often observed at the same range as the maximum cumulonimbus echo at the height. Taking account of beam width the echo received from 29,000 ft (range 65 mi) almost certainly originated above 25,000 ft, where the temperature was about — 29°C.

3. Echoes on 21 August

One of the PPI photographs taken on this day about the time when lightning was occurring is shown in Fig. 1 (Plate VI). It shows extensive echoes from cumulonimbus clouds; the echoes to north-west and south-west although of high intensity have the characteristic diffuse edges associated with the first stages of decay. Lightning echoes were received on a bearing of 289°Mag, which lies along the south-western edge of the echo to the north-west, and the height-range photograph taken on this bearing is shown in Fig. 2 (Plate VI). The angle at which the aerial was stopped is shown on this photograph and the remaining photographs are A-scope photographs taken on this bearing with this angle of elevation. The photographs were obtained by leaving the camera shutter open until a transient echo was seen and then immediately closing it. If no transient echo was seen in about 10-15 sec the shutter was closed and a new exposure begun. In this way photographs were obtained of the normal A-scope display of the quasi-permanent cumulonimbus echo within a few seconds on each side of the time of occurrence of the transient echo, enabling the transient echo to be identified with certainty. Thus Fig. 3 (Plate VI) is a 10-sec exposure which was completed without a transient echo being seen and was immediately followed by Fig. 4 (Plate VI), also a 10-sec exposure, in which transient echoes are visible at 45½ mi and from 49 to 53½ mi range. The photograph taken immediately after Fig. 4 was identical with Fig. 3 since no further transient echo occurred while it was being taken. In a similar way Figs. 5-8 (Plate VII) are photographs containing transient echoes and on each photograph the cumulonimbus echo, taken from the immediately preceding or following photograph, has been indicated by shading. In every case the transient echo was coincident with a lightning atmospheric on the R/T.

Allowing for the inclination of the radar beam and the vertical beam width of the equipment, the echoes at 40 mi probably originated between 17,000 and 23,000 ft, at 50 mi between 21,500 and 28,500 ft and at 53 mi between 23,500 and 30,500 ft. A height common to all ranges at which lightning was observed is therefore about 23,000 ft where the temperature (Crawley, 1500 GMT) was — 33°C. Cloud tops had almost certainly reached the tropopause at 28,000-29,000 ft, which was also the height of the echo tops, (see Fig. 2), the temperature being — 46°C.

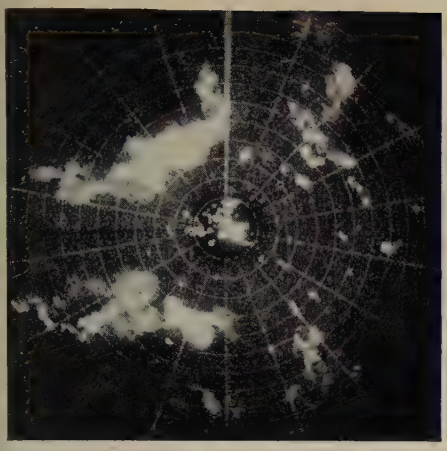

Figure 1. PPI photograph of radar echoes from thunderstorms – 21 August 1953, 1603·05 GMT. Range circles are at 5-mi intervals, the nearest visible range marker being at 10 mi. Radial lines are at 20° intervals from magnetic north.

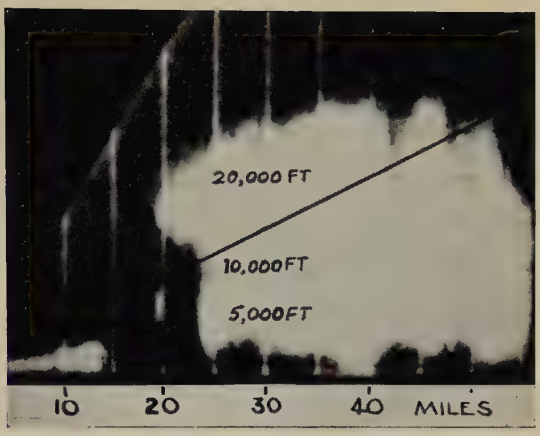

Figure 2. Height-range photograph of radar echo from the thunderstorms on a bearing of 289°Mag on Fig. 1. at 1600·50 GMT. The line sloping diagonally upwards from left to right shows the elevation angle on which the aerial was stopped for Figs. 3 to 8.

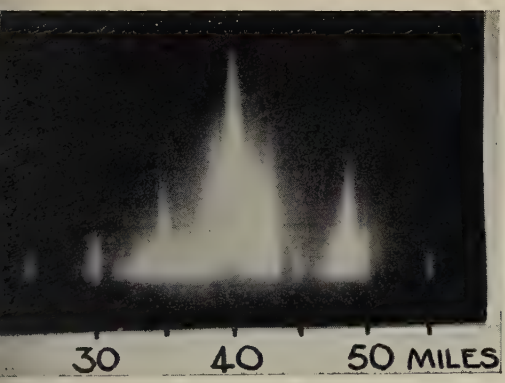

Figure 3. A-scope photograph of the echo shown in Fig. 2 along the line indicated at 1556·20-1556·30 GMT. Vertical displacement of the trace is proportional to the amplitude of the received signal.

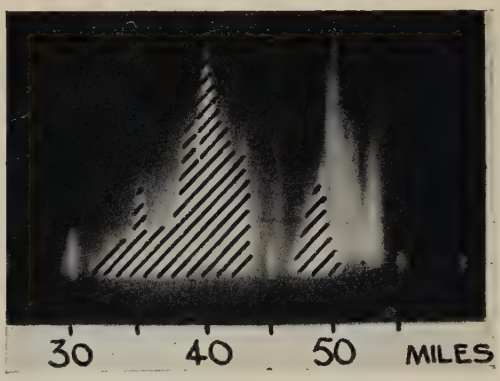

Figure 4. A-scope photograph as in Fig. 3 but at 1556·30-1556·40 GMT. As well as the thunderstorm echo, shown shaded, cf. Fig. 3, there is a lightning echo from 50-54 mi. The lightning echo is as strong as that from the thunderstorm but the brilliance is less owing to its short duration.

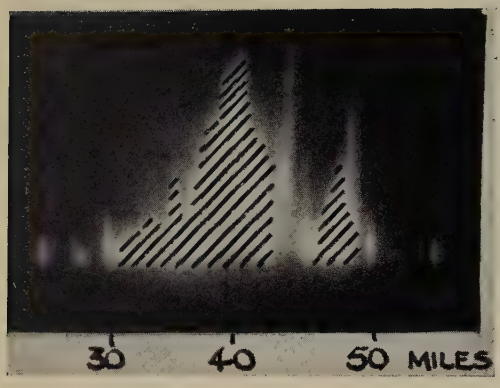

Figure 5. As Fig. 4, 1558·15-1558·45 GMT.

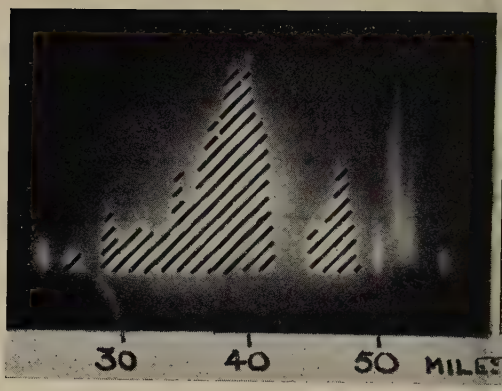

Figure 6. As Fig. 4, 1559·20-1559·30 GMT.

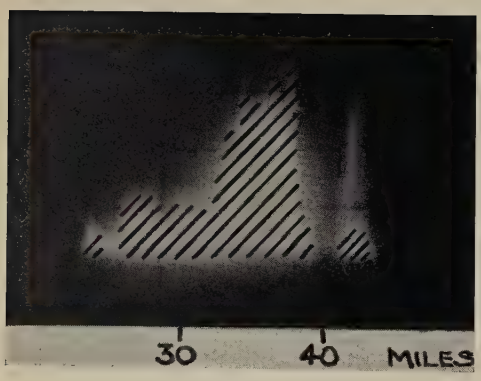

Figure 7. As Fig. 4, 1604·20-1604·30 GMT.

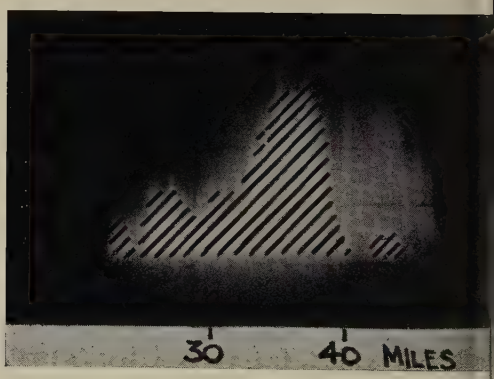

Figure 8. As Fig. 4, 1604·50-1605·05 GMT.

4. Points of interest

(i) No lightning echoes occurred on 12 and 21 August in the strongest part of the cumulonimbus echo at the height investigated, although in Figs. 7 and 8, lightning echoes were received from the region of the next maximum in echo intensity, probably from near the top of the second echo column at 45 mi on Fig. 2.

On the other hand on 20 August, when the echoes were smaller and more isolated, many flashes occurred in the region giving the strongest echo at the height investigated, although the technique used still made it possible that the echoes were received from close to the echoing volume rather than within it.

(ii) On many occasions flashes occurred in regions giving small or undetectable echo intensity but with the lightning echo extending right up to the boundary of a strong column (see, e.g., Figs. 4, 5 and 8).

(iii) Search for lightning echoes at lower heights than near to the top of the weather echo was not so successful.

(iv) Lightning echoes were received on each day from regions where the temperature was — 29°C or lower. In S. Africa, Miles (1953) noted the temperatures in the region of many of the lightning echoes as — 22·5°C to — 30°C while others were observed at temperatures of — 4°C to — 9°C.

(v) In Fig. 8 the lightning echo extends over about 9 mi and the flash must therefore have been almost horizontal (the angle of elevation of the aerial was about 4°).

(vi) It has been suggested by the writer (Jones 1950) that a contributory factor to the generation of a thunderstorm is the introduction of supercooled water into a region of ice crystals; this may occur at high levels in a cumulonimbus after anvil formation. The present evidence shows that lightning occurs at great heights within a cloud and after the cloud has reached its maximum vertical development.

5. Discussion of observations

Section 4 (ii) above implies that the lightning flashes often occurred within a volume of cloud where the particle size was below that normally associated with precipitation and, taking account of (iii), (iv) and (v), suggests that many flashes occurred within the anvil part of the cloud consisting principally of small ice crystals. Other work (Jones 1954) shows that the edge of a strong echoing volume is a region where an aircraft will usually experience severe turbulence and that the echo column itself is often coincident with an up-current. The fact that there is a strong echo implies the presence of large particles, either liquid or solid, and there is overwhelming evidence to suggest that there are within cumulonimbus clouds almost vertical columns having very different characteristics from the surrounding cloud. The present evidence suggests that many lightning strokes originate in the boundaries between these columns and the surrounding cloud, and hence that the hydrometeors in such columns are charged with opposite sign to the ice crystals in the anvil cloud through which the lightning flash is propagated. It is significant that Malan and Schonland (1951) have shown that in S. African thunderstorms the negative charge is often distributed throughout a nearly vertical column which may extend up to the — 40°C level. It seems at least an interesting possibility that the column of cloud and precipitation from which a very strong radar signal is received is negatively charged while the small ice crystals of the anvil cloud are positively charged. This is not at variance with Wormell (1953), who states that the process of selective ion capture envisaged by Wilson leads to most precipitation having a negative charge, with a preponderance of positive ions carried

upwards to the top of the cloud (and thence presumably into the spreading out anvil), provided it is realized that ' precipitation ' can also be carried upwards within the strong upward currents of the thunderstorm to great heights.

Acknowledgment

I am indebted to the Director of the Meteorological Office for permission to publish this paper.

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551.574.1

The concentration, size distribution and production rate of large salt nuclei over the oceans

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(Manuscript received 5 May 1954, in revised form 26 July 1954)

Summary

The sampling and measurement of large condensation nuclei (equivalent salt contents greater than 2×10^{-14} g) on board of Weather Ships in the N. Atlantic, 200 mi from land, reveal the existence of two distinct types of size spectra. It seems that the nuclei which predominate when waves are breaking (Type I distribution) are the residues of spray droplets, but that those which occur with wind speeds of less than 7 m sec^{-1} have a (Type II) distribution which closely resembles that of continental aerosol. In winds of up to 15 m sec^{-1} the measured concentrations of large salt nuclei rarely exceeded 10/cm^3.

From the measured concentrations and size distributions of Type I nuclei, it is deduced that in winds of 15 m sec^{-1} the rate of production at the sea surface of salt nuclei with mass greater than 10^{-13} g is about 40 cm^{-3} sec^{-1}; very similar values have been obtained from laboratory experiments in a wind-wave tunnel. The corresponding rate for nuclei with $m > 2 \times 10^{-14}$ g would be about 100 cm^{-3} sec^{-1}.

Laboratory studies show that the bursting of air bubbles of diameters between 0·3 mm and 4 mm at the surface of sea water produces small numbers of rather large drops (diameters 50-500 μ), the majority of which would fall back quickly into the sea. The large salt nuclei found in the atmosphere are probably the residues of smaller drops produced by disintegration of the bubble caps.

1. Introduction

It is well known that nuclei of radius greater than about 10^{-5} cm which escape detection in the Aitken Counter are thrown into the air by the disintegration of sea water and have been found by Woodcock (1952) up to heights of 3,000 m on occasions when the lower atmosphere was unstable. Woodcock (1952) and Moore (1952) have found sea-salt particles of mass greater than 10^{-13} g in concentrations of up to about 10 cm^{-3} at about 15 m above the sea surface and both have found these nuclei in increasing numbers with strong winds.

The results of some preliminary measurements of the size distribution of large hygroscopic nuclei collected on board an Ocean Weather Ship in the North Atlantic were reported by Moore (1952). These measurements have since been extended to include nuclei of crystal radius greater than 10^{-5} cm and will now be described. From these measurements an estimate is made of the rate of nucleus production at the sea surface, an independent assessment of which is obtained from laboratory studies in a wave tunnel.

2. The concentration and size distribution of large nuclei

The collectors used were glass strips coated with a hydrophobic silicone film. In strong winds they were exposed on a wind-vane, and at low wind speeds they were fixed across the intake of a suction pump. The results obtained when both methods were used simultaneously in strong winds were in excellent agreement. Using strips of width 0·25 mm, nuclei of mass greater than 2×10^{-14} g were captured. The collectors were examined under the microscope in a chamber whose walls were maintained at constant temperature and which contained a 10 per cent solution of NaCl. Measurements of the equilibrium radii of the hemispherical droplets allowed computation of thenuc lear mass m from the equation $m = 0{\cdot}1 \times 2\pi r^3/3$ assuming the nuclei to consist of sodium chloride.

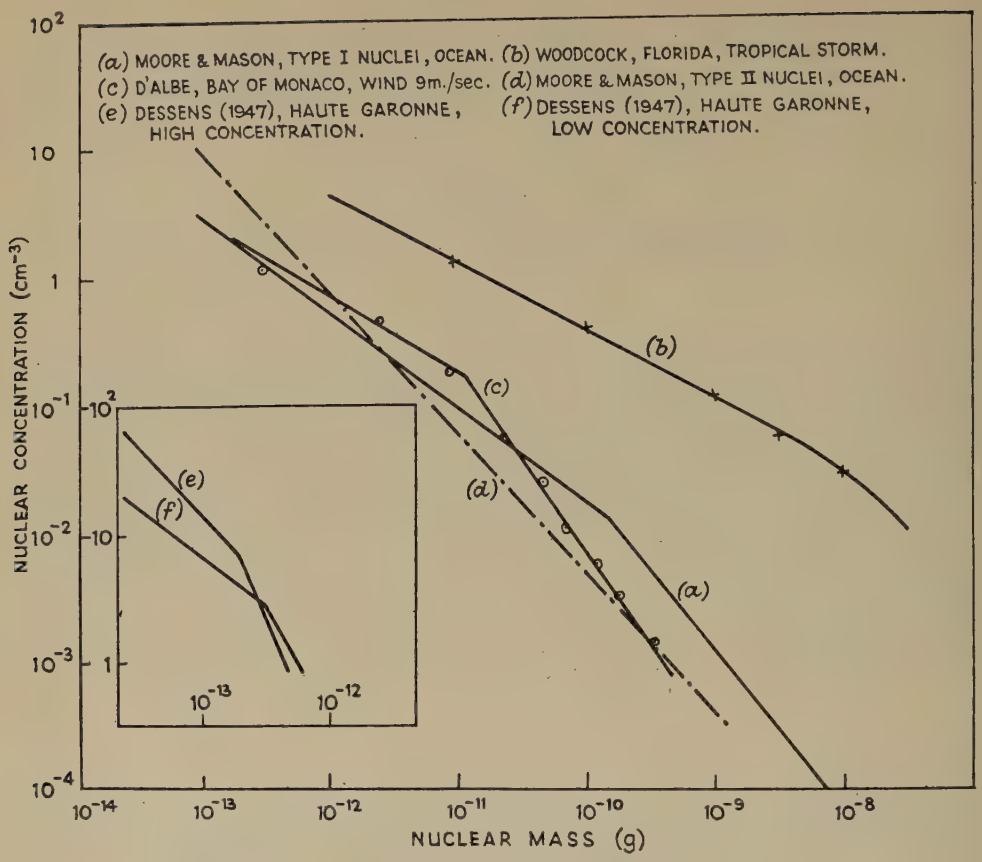
(a) MOORE & MASON, TYPE I NUCLEI, OCEAN.
(b) WOODCOCK, FLORIDA, TROPICAL STORM.
(c) D'ALBE, BAY OF MONACO, WIND 9m./sec.
(d) MOORE & MASON, TYPE II NUCLEI, OCEAN.
(e) DESSENS (1947), HAUTE GARONNE, HIGH CONCENTRATION.
(f) DESSENS (1947), HAUTE GARONNE, LOW CONCENTRATION.
NUCLEAR CONCENTRATION (cm⁻³)
NUCLEAR MASS (g)

Figure 1. The size distributions of nuclei over land and sea.

The size distributions of these large nuclei could be classified into two types. A Type I distribution (Fig. 1 (a)) obeys a law of the form $N = A_1/m^{b_1} = B_1/r^{3b_1}$, where N is the number of nuclei of mass greater than m and radius greater than r, $b_1 = 0{\cdot}75$ for $m < m_c$ and takes values of 1·4 upwards when $m > m_c$, where m_c is the nuclear mass at which the discontinuity occurs. The value of m_c varies from 10^{-11} g to greater than 10^{-8} g as the wind speed increases from 6 m sec^{-1} to more than 15 m sec^{-1}. A very similar distribution was obtained by Fournier d'Albe (1951) working over the Bay of Monaco in winds of 9 m sec^{-1} (Fig. 1 (c)), while the size distribution of the smaller nuclei ($m < m_c$) are matched by Woodcock's (1950) curve for the nucleus distribution in a tropical storm off the coast of Florida (Fig. 1 (b)). The concentrations of the largest of these nuclei showed an increase with increasing wind speed when the latter exceeded 7 m/sec and white caps were forming. With a wind speed of 15 m/sec, the maximum concentration of nuclei with $m > 2 \times 10^{-14}$ g was found to be 10/cm^3.

A distinctly different type of distribution curve (Type II) was also observed, but only at wind speeds less than 7 m sec^{-1}. A typical curve is shown in Fig. 1 (d); it shows no discontinuity over the measured range of nuclear sizes (10^{-13} g $< m < 10^{-8}$ g) and can be represented by the formulae $N = A_2/m = B_2/r^3$. The concentrations of these Type II nuclei were somewhat higher than those of Type I even when the latter had been sampled in high winds.

Type I distributions were found on 25 occasions and Type II distributions on only 5 occasions.

For Type I nuclei with $m > m_c$, the slope of the mass distribution curve is found to vary with the surface wind speed; in general, it is steeper for lower wind speeds, increasing from about 1·4 to about 2·1 over the range 15 m sec^{-1} to 6 m sec^{-1}, and steeper also when the atmosphere is more stable. Now, strong winds will produce greater numbers of nuclei at the sea surface and also distribute them throughout a greater depth of atmosphere. On the other hand, the large nuclei are lost by sedimentation and more readily in stable conditions. It appears that the presence of a discontinuity or a sharp change of slope in the Type I distribution curves can be explained in terms of a loss of the larger nuclei by sedimentation. In strong winds, the part of the curve to the right of the discontinuity probably represents a state of dynamic equilibrium between the rates of production and loss of the larger nuclei. In light winds, when the loss rate of the larger nuclei by sedimentation will be greater than their rate of production, we should expect this part of the curve to have a considerably steeper slope than in strong winds.

Our Type I distributions are very similar to those of Woodcock but he does not appear to have encountered Type II distributions over the sea. It may well be significant that the Type II distributions, which were found only when the wind speed was less than 7 m sec^{-1}, (i.e. in the absence of white caps), and therefore when the current production of sea-salt nuclei was probably small, obeyed the same r^{-3} law which Junge (1953) has found for continental aerosol and which also holds for aerosol collected in our London laboratory. This suggests that although some salt particles were identified among the Type II nuclei, they may have been considerably reinforced by hygroscopic nuclei of land origin particularly as the concentrations of Type II nuclei were rather higher than those of Type I. Over Hawaii, where Woodcock made most of his measurements, continental aerosol may have been relatively absent.

Dessens (1949), working in the open country, has found haze-droplets in mean concentrations of 200 cm^{-3} and believes them to consist of sea-salt solution. However, he has been able to identify sodium-chloride crystals in only the largest droplets comprising about 1 per cent of the total. It is difficult to avoid the conclusion that most of his smaller droplets must have been of land origin, since his concentrations of nuclei with equivalent salt mass greater than 3×10^{-14} g were at least three times greater than our highest measured values over the ocean in strong winds. Some support is given to this suggestion by a study of Fig. 1 (e, f), which shows that on those occasions when Dessens' nucleus concentrations were highest, his smaller nuclei ($m < m_c$) obeyed a Type II (r^{-3}) distribution, but on those days when the concentrations were low they more closely resembled a Type I distribution.

3. Rates of production of large nuclei at the sea surface

(*a*) *A laboratory study of large-nucleus production by breaking waves*

Breaking waves were produced in a wind tunnel, 6 m long, 7·6 cm wide and 38 cm high containing sea water to a depth of 15 cm. The wind speed in the tunnel could be varied to reproduce the range of conditions which obtain over the oceans when the wind speed measured at a height of 10 m varies from 0 to 25 m sec^{-1}. The nuclei were caught on hydrophobic glass slides placed 17 cm above the undisturbed water surface and 5 m from the windward end of the tunnel. Measurements were largely restricted to equivalent wind speeds of less than 20 m sec^{-1}, since with stronger winds the collectors were covered with large spray produced by shearing off of the wave crests. The concentrations of salt nuclei with $m > 10^{-13}$ g were found to be a maximum in winds of between 16 m sec^{-1} and 18 m sec^{-1}; stronger winds produced an increase only in the numbers of the very large spray drops which soon fell back into the water.

An estimate was made of the rate of production of the nuclei with $m > 10^{-13}$ g by assuming that the nuclei present at wind speeds below 6·4 m sec^{-1} (the velocity at which air bubbles first appeared in the sea-water) were originally present in the air, and that any increase in concentration at higher wind speeds was due to sea-spray nuclei. It was impossible to detect any changes in the concentration of smaller nuclei due to the breaking waves because of the very high natural concentrations of small nuclei in London air.

With a wind speed of 8 m sec^{-1} in the tunnel, equivalent to 16 m sec^{-1} over the sea at 10 m, the rate of nucleus production Q as calculated from the relation $Q = NUz/x$ where N is the measured concentration, U the wind velocity at collector-level, z the depth of the air space above the water and x the distance which the air had travelled over the producing surface, was found to be 40 cm^{-2} sec^{-1} for nuclei of $m > 10^{-13}$ g.

(*b*) *Estimation of rate of nucleus production from size distributions observed over the sea.*

We shall assume that in the Type I nucleus distributions at high wind speeds (e.g. Fig. 1 (a)), for nuclei of $m > m_c$, equilibrium existed between their rate of production and rate of sedimentation, but that the smaller nuclei were being produced more rapidly than they settled out. On this basis, we now proceed to calculate the rate of production of the nuclei in terms of their measured concentrations.

For the larger nuclei ($m > m_c$) we may write

$$q_1\, dr = v_1 n_1(o) dr \quad . \quad . \quad . \quad . \quad (1)$$

where $q_1\, dr$ is the rate of production of nuclei with radii between r and $r + dr$, v_1 is their fall velocity and $n_1\,(o)\, dr$ their concentration at the surface. For the smaller nuclei ($m < m_c$), we have

$$q_2\, dr = \gamma\, n_2\,(o)\, dr \quad . \quad . \quad . \quad . \quad (2)$$

or

$$Q_2 = \gamma\, N_2 \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

where γ is a parameter having the dimensions of velocity whose magnitude is determined by the properties of the air flow but is independent of nuclear size, Q_2 is the production rate of all nuclei with $r < r_c$, their concentration being N_2.

The concentration of all nuclei with $r > r_c$ is

$$N_1 = \int_{r=r_c}^{\infty} n_1\,(o)\, dr = \int_{r=r_c}^{\infty} q_1/v_1\, dr \quad . \quad . \quad . \quad (3)$$

At the highest wind speeds encountered (15 m sec^{-1}) we have already seen that $N_1 = B_1/r_c^{4\cdot2}$, and v_1 may be expressed in the form $v_1 = 1{\cdot}3 \times 10^6 f^2\,(H)\, r^2$, where $f(H)$ is a function which expresses the change of nucleus droplet size with relative humidity H.

Thus the two expressions for N_1 are identical if

$$\frac{d}{dr}\left(\frac{B_1}{r^{4\cdot2}}\right) = \frac{q_1\, dr}{1{\cdot}3 \times 10^6 f^2\,(H)\, r^2} \quad . \quad . \quad . \quad (4)$$

The rate of production of the larger nuclei is then given by

$$Q_1 = \int_{r=r_c}^{\infty} q_1\, dr = -\,1{\cdot}3 \times 4{\cdot}2 \times 10^6\, B_1 \int_{r=r_c}^{\infty} \frac{f^2\,(H)\, dr}{r^{3\cdot2}} \quad . \quad (5)$$

or

$$Q_1 = \frac{4{\cdot}2}{2{\cdot}2} N_1\, v_c \quad . \quad . \quad . \quad . \quad (6)$$

Now when $r = r_c$, $Q_1 = Q_2$, $N_1 = N_2$, $v_1 = v_2 = v_c$ and $\gamma = Q_2/N_2 = Q_1/N_1 = 1{\cdot}9\ v_c$, from which γ can be evaluated.

In winds of 15 m sec^{-1}, the value of m_c was found to be 10^{-8} g, the corresponding fall speed v_c in an atmosphere of 80 per cent relative humidity being 4·5 cm sec^{-1}. Under these conditions the value of γ turns out to be 8·6 cm sec^{-1}.

The rate of production of the smaller nuclei under the same conditions can now be calculated from the equation $Q_2 = \gamma N_2 (o)$. The maximum observed concentrations of nuclei with $m > 2 \times 10^{-14}$ g was 10 cm^{-3}, indicating a rate of production of 86 cm^{-2} sec^{-1}. Nuclei of $m > 10^{-13}$ g were found in concentrations of 5 cm^{-3} with winds of 15 m sec^{-1} the corresponding production rate of 43 cm^{-2} sec^{-1} being in very good agreement with that observed in the wave-tunnel experiment.

4. The production mechanism of large nuclei

The production of salt nuclei by the disintegration of sea-water was studied in the laboratory by Köhler (1941). He caused an air jet to impinge tangentially on to the surface of water enclosed in a glass tube and deduced the mass, radius and density of the droplets produced, from their motion in a combined gravitational and electric field. He deduced that droplets containing between 2×10^{-14} g and 10^{-12} g of salt were produced together with many too small to study with his technique.

The mechanism by which the largest salt particles are produced is suggested by some high-speed photographs of bursting bubbles recently published by Woodcock, Kientzler, Arons and Blanchard (1953) and by Knelman, Dombrowski and Newitt (1954a). It appears that, after bursting of the bubble cap, liquid rushes into the crater to form an unstable jet and this breaks up to produce one or more drops which are projected into the air.

In the present investigation the number and size of the drops produced by bursting bubbles, and the mass of salt contained in them, have been determined. The apparatus consisted of a large inverted bell-jar containing sea-water with an air jet entering vertically from beneath and releasing single air bubbles at a controlled rate 5 cm below the water surface. The bubble diameter was varied between 0·33 mm and 4·3 mm by using jets of different diameters. The top of the jar was either covered with a glass plate coated with a hydrophobic silicone or enclosed by a cylindrical brass box clamped to the rim of the jar and in the side of which was a tube containing a holder for a suitably coated glass microscope-slide mounted on a travelling arm (Fig. 2). The slide could be pushed out of the side tube and exposed at the desired height above a bursting bubble and then retracted into the side tube for microscopic examination. The larger drops were caught on the glass plate and measured *in situ* under the microscope while the smaller ones were also caught on the microscope slides coated with either magnesium oxide or oil. In the enclosed experimental space, the relative humidity was assumed to be 98 per cent, the equilibrium value over sea-water.

The average diameters of the drops produced by bursting bubbles of various sizes, as measured at a height of 5 cm above the water surface (except in the case of the smallest bubbles when the drops did not reach this level), were as shown in Table 1; there was remarkably little spread in the size of the drops produced by bubbles of a given diameter.

These drop diameters are larger than those given by Boyce (1953)* by a factor of about 2·5 but his values represent equilibrium diameters for an atmosphere of 83 per cent relative humidity whereas ours were measured under a relative humidity of 98 per cent; this would account for the two sets of results being different by a factor of two.

* The authors' attention was drawn to the paper by Boyce after this paper had been submitted for publication.

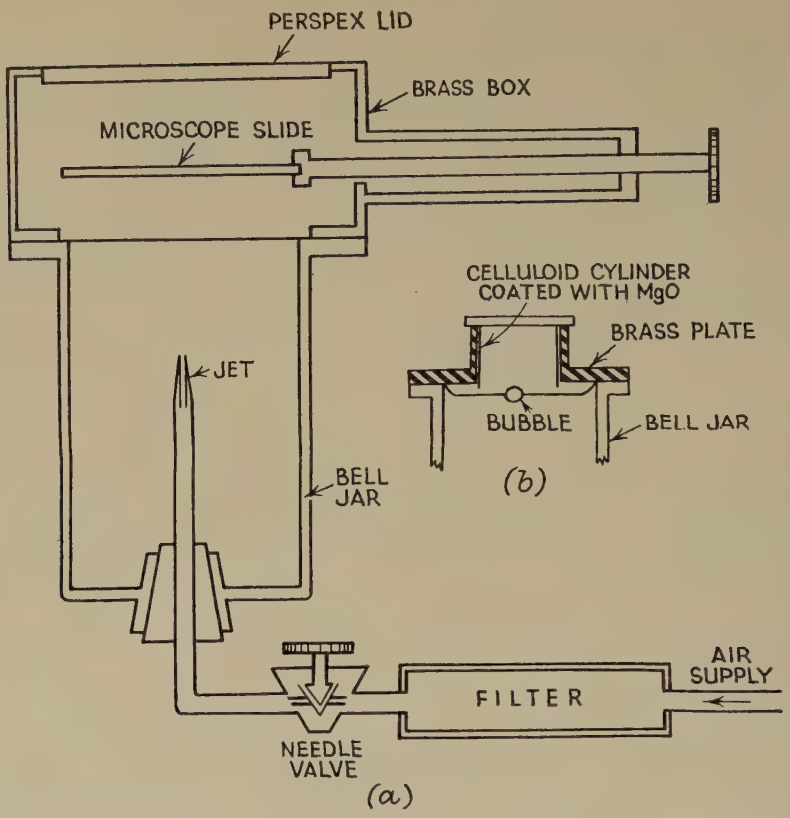

Figure 2. Apparatus for measurement of drops produced by bursting bubbles.

TABLE 1. MEAN SIZE OF DROPS PROJECTED VERTICALLY TO A GIVEN HEIGHT BY BURSTING BUBBLES OF VARIOUS DIAMETERS

Bubble diameters (mm)	Mean drop diameter (μ)	height (cm)
4·3	550	5
3·4	430	5
2·0	270	5
1·0	190	5
0·3	65	2

These large drops were projected vertically upwards and doubtless resulted from disruption of the liquid jet. Bubbles of about 2 mm diameter sometimes projected drops to heights of nearly 20 cm but the maximum height attained fell off as the bubble size was increased or reduced, a result which was also obtained by Boyce.

These measurements suggest that, if the majority of bubbles produced by breaking waves are larger than about 0·5 mm diameter, most of the drops formed by this jet mechanism will fall back quickly into the sea and not contribute to the supply of potential condensation nuclei, although the smallest droplets, of diameter about 100 μ, containing about 10^{-8} g of salt and having fall-speeds of about 25 cm sec^{-1}, might be carried up in strong winds. Nuclei of this size were found in concentrations of about 100/m^3 by us at 10 m above the surface, and in smaller concentrations by Woodcock (1952) at heights

of several hundreds of metres over Hawaii. If, however, large numbers of appreciably smaller bubbles are formed, the drops resulting from disruption of the jets, presumably being smaller than those from larger bubbles, might make a significant contribution to the giant nuclei.

But in addition to the very large spray drops produced by disruption of the liquid jet, it seems that smaller drops are produced by disintegration of the bubble film. Since most of these will evaporate very quickly if thrown into an atmosphere of, say, 80 per cent relative humidity, it was necessary to catch them in an enclosed chamber where the air was saturated with respect to sea-water. The droplets were caught on either magnesium-oxide-coated slides arranged to form the sides of a square box surrounding the bursting bubbles or on the inner surface of a cylinder made from thin celluloid strip coated with magnesium oxide see Fig. 2 (b). Since most of the droplets from the bursting bubble were projected at a low angle (about 10°) to the horizon it was necessary to have the lower edges of the collecting surfaces very close to the water surface. The formation of a small depression in the water surface by a small central probe ensured that most of the bubbles burst in the centre of the collecting system and not at the boundaries where they would ruin the oxide film.

The great majority of droplets found on the slides had diameters between 30 μ and 5 μ and contained between 5×10^{-10} g and 2×10^{-12} g of salt. Bubbles of diameter about 3 mm produced, on the average, about one droplet of diameter greater than 10 μ. Larger bubbles tended to produce larger numbers of rather smaller droplets, which is consistent with the fact thay they possess thinner films. These results have been confirmed in an independent investigation by Knelman *et al.* (1954b).

Droplets of smaller diameter than could be measured with the magnesium-oxide slides were also formed; examination of the hydrophobic slides revealed that nuclei of mass down to 2×10^{-14} g were produced by the bursting bubbles. There were almost certainly even smaller particles which could not be detected by these techniques.

5. Conclusions

It appears that sea-salt nuclei of mass greater than 2×10^{-14} g, the study of which is the subject of the first part of this paper, are produced by the bursting of air bubbles formed by the trapping of air in breaking wave crests. The very largest nuclei ($m > 10^{-9}$ g) are produced by the disintegration of a liquid jet formed in the cavity left by a bursting bubble and the rest by the shattering of the bubble film. The concentrations of these large and giant nuclei (mass range 2×10^{-14} g to 10^{-8} g) rarely exceed 10 cm^{-3} even when the sea is extremely agitated and their rate of production averaged over the wave-breaking area is probably not much in excess of 100 cm^{-2} sec^{-1}. It seems, therefore, that these nuclei are not sufficiently numerous nor are they produced at a sufficient rate to be a major source of the nuclei involved in cloud formation.

It may well be that much larger numbers of nuclei, too small to be detected by the methods described in this paper, are produced by the bursting of bubble films. Some evidence in support of this has been obtained in some preliminary experiments in which the bursting of air bubbles in sea-water has been studied in a cloud chamber, the small nuclei being made visible by condensation. The nature, size and rate of production of these small nuclei are now being investigated.

The measurements made on the Weather Ships suggest that large nuclei of land origin may be transported for considerable distance over the sea and, at distances of some 200 mi may be predominant on occasions when the wind speed is less than 7 m sec^{-1} and the current production of sea-salt nuclei is small.

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Secular changes in the rainfall regime of SE. Australia

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Summary

Analysis of long-series rainfall records from Victoria and New South Wales shows a decrease of summer rainfall to a minimum about the turn of the century, and fifty years' gradual increase since then. Winter rainfall trends were opposite. Fluctuations of a much larger amplitude affected the rainfall regime of autumn and spring, suggesting possibly a discontinuous change of the climatic development about 1893.

The winter rainfall of southern New South Wales is positively correlated with the mean strength of the westerlies at 300 mb. The correlation is negative in summer. It is concluded that the changes of the rainfall regime were associated with an increase of intensity of the upper westerlies to a maximum at about 1900 and a following decrease. During the period of maximum mean westerly flow the formation of east-coast cyclones in autumn and spring may have been inhibited and the monsoon rains of tropical Queensland were below normal.

1. Introduction

The existence of climatic changes that are effective within a human lifetime has been demonstrated by the study of glaciers, lakes, trees and of meteorological records. These changes must have been associated with some variation in the mean atmospheric motion. The character of such an association may be expected to be simpler in large parts of the southern hemisphere than in the northern. Australia, in particular, offers rather favourable conditions for its study. It is a fairly flat island continent, with surrounding oceans that are free from large zonal temperature differences. The meteorological records available are at the same time of a comparatively high standard.

Geographically the present analysis is limited to rainfall data from the southern half of New South Wales and from Victoria. The Great Dividing Range runs parallel to the coast through this region. It has a mean height of more than 6,000 ft in the Snowy Mountains which stretch from the source of the Murray in the south to Kiandra in the north. Fig. 1 shows the lie of the high ground and the stations whose rainfall records were investigated.

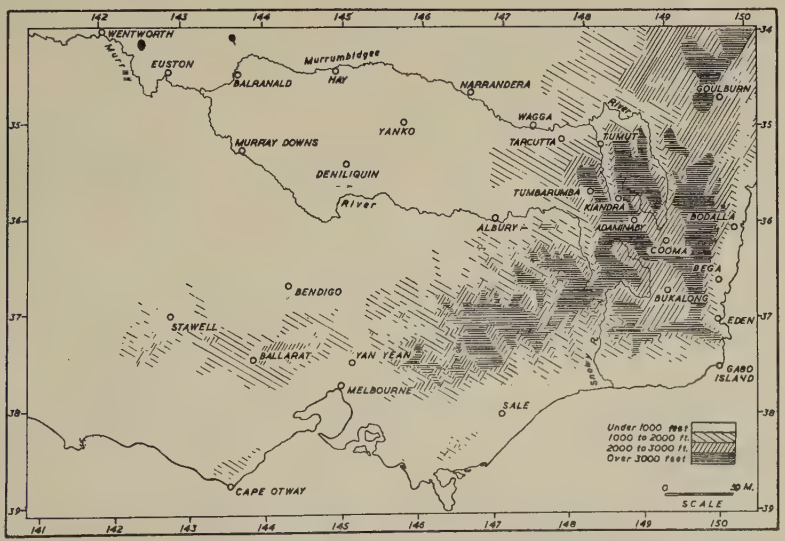

Figure 1. Location of rainfall-stations.

The area shows great diversity in its rainfall. The mean annual precipitation varies from 11 in. in the western Murray plains to about 100 in. on parts of the Snowy Mountains and more than 60 in. in some coastal districts. The rainfall regime shows, in general, a pronounced winter maximum and summer minimum both in the dry western plains and on the Snowy Mountains. To the east of the mountains there are two maxima, in summer and winter, and often a third one in October. The highest maximum however occurs there usually in summer. The reason for the difference between east and west will be discussed later.

Changes in the atmospheric circulation should have different effects on the climate in different seasons, nor can it be expected that the same cause will necessarily affect the circulation in the same way in summer and winter. For this reason the various seasons were analysed separately.

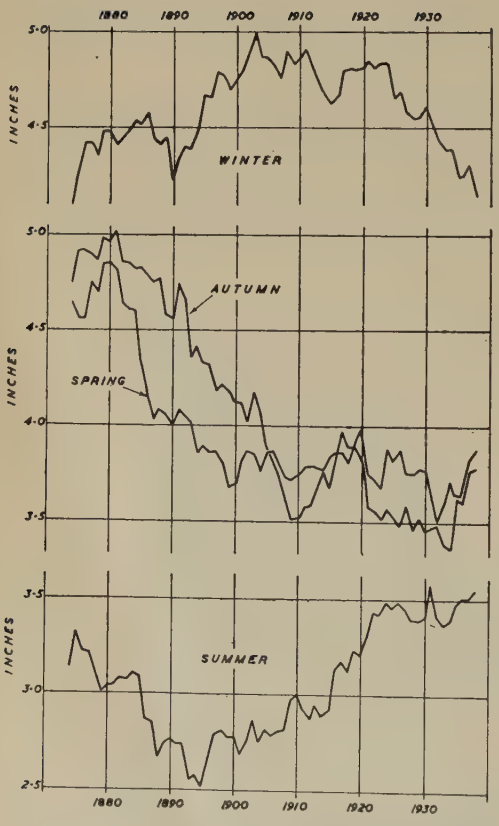

Figure 2. Thirty-year running averages of rainfall at Deniliquin.

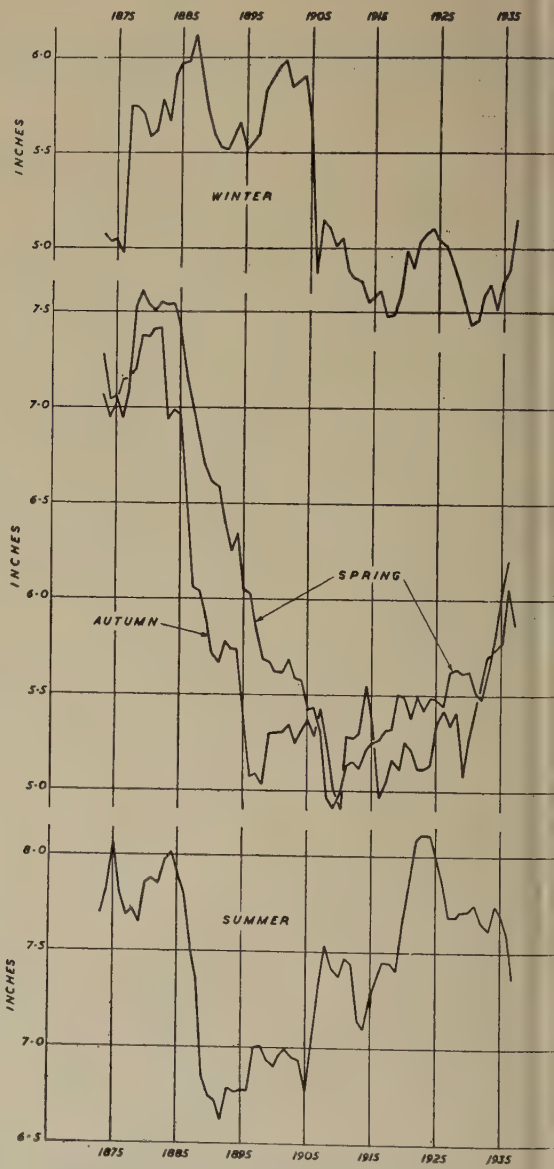

Figure 3. Thirty-year running averages of rainfall at Bukalong.

2. Fluctuations of 30-year seasonal averages

In order to give a general impression of the changes which have occurred, 30-yr running seasonal averages are shown in Figs. 2 and 3 for Deniliquin and Bukalong. Rainfall records have been kept in Bukalong since 1858 and in Deniliquin since 1859, which makes them the two oldest stations still existing in southern New South Wales. The averages are plotted against the 16th year of the thirty-year interval.

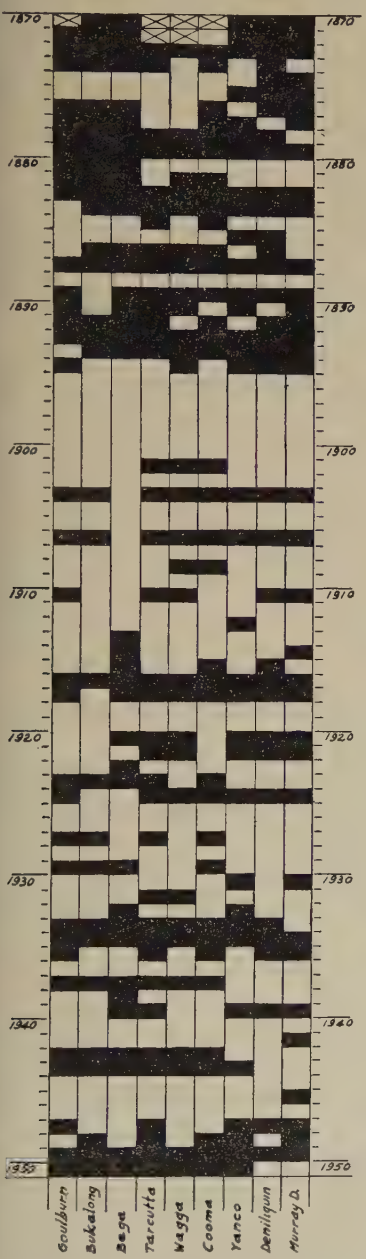

Figure 4. Spring rainfall in southern New South Wales. Springs with rainfall above average marked dark.

In summer (January, February together with December of the preceding year), both diagrams show a gradual rise of the average rainfall over the past fifty years. In Deniliquin this rise is evident, from the thirty-year period centred on 1895, onwards. In Bukalong it started slightly earlier. This is in agreement with E. L. Deacon's (1953) comparisons of rainfall in the period 1881-1910 with rainfall 1911-1940. Deacon concluded that the summers in SE. Australia tend to become wetter. Figs. 2 and 3 would suggest that before this trend began, the mean summer rainfall decreased to a minimum which more or less coincides with Deacon's period I.

The movement of the winter (June, July, August) running average is more irregular but may be said to be roughly opposite to the summer curve. The gradual decline over the last fifty years supports once more Deacon's conclusion that winters tend to get drier.

The most striking part of Figs. 2 and 3 is, however, the extremely marked decline of the average rainfall in autumn (March, April, May) and in spring (September, October, November). In the case of Bukalong this decline amounts to more than one-third both for autumn and spring rains. It is not peculiar to the two stations but can be traced everywhere in New South Wales and eastern Victoria where sufficiently long records are available, representing hence a major change in the Australian climatic pattern.

3. Discontinuous climatic change

The use of running averages obscures the details of the climatic change. If the data are analysed individually, it appears that something like a discontinuous break in the rainfall regime occurred about 1893-94. Fig. 4 shows strikingly that, before 1893, spring rains tended to be above their long-term mean for most years and for long periods of time. This was followed by a prolonged dry spell in the early twentieth century and a slow return to wetter conditions since then. The pattern for autumn is similar.

A more quantitative representation is given by Table 1. Twenty-five year means were chosen in this case because of the scarcity of stations with records prior to 1869. The difference between the means for the 25 years prior and subsequent to 1893 can be shown to be significant on the 1 : 1,000 level.

TABLE 1. MEAN VALUES OF SPRING RAINFALL (INCHES) FOR SUCCESSIVE 25-YEAR PERIODS

	Bukalong	Yanko	Deniliquin	Cooma	Goulburn	Sydney	Melbourne
1869-1893	8·24	5·00	4·77	6·42*	7·43*	10·02	7·98
1894-1918	4·78	3·53	3·63	4·30	5·40	7·85	6·52
1919-1943	5·67	3·66	3·60	4·57	6·22	8·47	—

Except for Deniliquin, all stations show a return to wetter springs in the third period. Autumn records present a similar variation.

The difference in spring and autumn rainfall necessarily affected the yearly totals. The contrast in the yearly means is, however, nowhere nearly as marked as it is for spring.

TABLE 2. MEAN VALUES OF YEARLY RAINFALL (INCHES) FOR SUCCESSIVE 25-YEAR PERIODS

	Bukalong	Yanko	Deniliquin	Cooma	Goulburn	Sydney	Melbourne
1869-1893	29·49	17·69	17·49	19·95*	26·40*	51·32	26·30
1894-1918	21·58	14·86	15·28	17·65	22·98	43·93	24·96
1919-1943	23·88	15·64	14·87	18·64	24·70	43·34	—

* 23-year mean

The sudden character of the change suggests that climatic variations are not necessarily a continuous process.

The mean position of the westerlies could conceivably alter in a discontinuous manner from one group of years to another. Alternatively, even a gradual variation of the circulation pattern might produce an apparently discontinuous change in a marginal phenomenon such as frontal spring-rainfall in Australia. To elucidate these problems, it is desirable to obtain quantitative correlations between mean rainfall and the upper atmospheric circulation.

4. RAIN AND AIR CIRCULATION OVER SE. AUSTRALIA

The winter rains which provide the southern half of Australia with most of its water are associated with frontal systems, troughs and depressions which move with the westerly air current of the southern hemisphere. The vertical motion necessary to form cloud or rain derives its energy largely from the westerlies, that is, ultimately, from the meridional heat exchange.

Summer rains on the other hand – not only in the tropical monsoon region proper, but also on the rest of the continent – are associated almost invariably with local instability. The ascending motion derives much of its energy from the heated continental surface, though the degree of instability will be determined also by the general circulation pattern.

In the west and the centre of the continent, the two regimes are separated by an arid belt with a region of dry summers in the south and dry winters in the north. In the east however, they merge into each other both geographically and in time. An analysis of surface synoptic situations and rainfall, extending over the three years 1951-53, suggests that up to 90 per cent of the autumn and spring rainfall along the coast of New South Wales, south of Sydney, and on the adjacent highlands was associated with east-coast cyclones. These are depressions with an indistinct or non-existing frontal structure. In autumn they frequently originate as tropical cyclones further north. In spring they may appear at first as frontal depressions in the westerlies. Sometimes they form in situ. In all cases there is, however, a tendency for the depression to travel slowly in a southerly direction, or to linger over the south-eastern part of the continent or the adjacent parts of the Tasman Sea.

The energy of these east-coast cyclones appears to be derived partly from the general circulation and partly from local instability and monsoonal effects. The analysis of one particular storm which gave heavy rains in October 1952 has been published elsewhere (Hanay and Kraus 1954). In that case the development was associated with vertical instability in a moist, almost stagnant air mass in the lower troposphere and the arrival of a trough or wave in a strong westerly current above.

In winter the distribution of the rains is more strongly affected by the orographic configuration than appears to be the case in other seasons. The rain-bearing west winds of winter are comparatively stable and rain forms at a low altitude. This results in very heavy winter precipitation on the western versant and the crest of the range. The eastern slopes, by contrast, suffer from a very pronounced Föhn effect. The mean August rainfall in Cooma, for example, is only 0·97 in. which is about ten times less than on the range, 50 mi to the west. In the warm seasons this contrast is to a considerable extent evened out. Rain has then no longer a predominantly western origin. Even if it does approach from that direction it will have generally a more unstable showery character, with a freezing level which is high above the hills and with a correspondingly high level of rain formation, causing the Föhn effect to become less pronounced.

5. Upper-air data

Regular radio soundings have been carried out since 1944 at Laverton (37° 52′S, 144° 46′E) to the south of the region and at Rathmines (33° 03′S, 151° 36′E) to the north. There is comparatively little indication of a relation between mean seasonal rainfall and the zonal wind at the 700-mb or the 500-mb pressure level. This could be expected as winter rainfall is associated with troughs and depressions, that is, with partly meridional flow. At 300 mb, however, a strong correlation emerges.

At the height of the 300-mb surface the mean wind over southern New South Wales is predominantly zonal. A difference, between the two stations, of 100 geodynamic metres in the mean height of an isobaric surface thus corresponds to a mean westerly of 14·2 m/sec or of 27·5 kt.

Table 3 gives the mean seasonal geodynamic-heights of the 300-mb surface over Laverton and Rathmines, the difference $\Delta\phi$ between the heights, the corresponding zonal mean wind $\overline{U}$ and, for reasons discussed in the following sections, the ranking of $\Delta\phi$ or $\overline{U}$ according to magnitude from the lowest to the highest value. The mean seasonal heights were not computed if the number of observations during any one month was less than twelve. The means are hence based on 36 to 90 individual observations. As these were not always taken simultaneously at Laverton and Rathmines, the values of $\Delta\phi$ and $\overline{U}$ are necessarily approximate.

TABLE 3. MEAN SEASONAL 300-mb DATA FOR LAVERTON AND RATHMINES

	Laverton geod. m	Rathmines geod. m	$\Delta\phi$	$\overline{U}$ m/sec	Rank	Laverton geod. m	Rathmines geod. m	$\Delta\phi$	$\overline{U}$ m/sec	Rank
	Summer					Autumn				
1945	9317	9460	143	20·2	6	9315	9384	69	9·8	5
1946	9507	—	—			9282	9369	87	12·4	6
1947	—	9484	—			9328	9366	38	5·4	2
1948	9353	—	—			9296	9342	46	6·5	3
1949	9379	9461	82	11·6	3	9351	9386	35	5·0	1
1950	9406	9463	57	8·1	2	9314	9425	111	15·8	9
1951	9456	9504	48	6·8	1	9252	9348	96	13·6	8
1952	9406	9506	100	14·2	5	9278	9366	88	12·5	7
1953	9405	9488	83	11·8	4	9359	9420	61	8·7	4
	Winter					Spring				
1945	9105	9232	127	18·0	7	9249	9326	77	10·9	3
1946	—	9175	—			9238	9281	43	6·1	1
1947	9063	9156	93	13·2	4	9206	9275	69	9·8	2
1948	9119	9204	85	12·1	2	9181	9269	88	12·5	4
1949	9125	9186	61	8·7	1	9192	9307	115	16·3	9
1950	9118	9215	97	13·8	5	9236	9348	112	15·9	7·5
1951	9084	9212	138	19·6	8	9207	9304	97	13·8	5
1952	9123	9210	87	12·4	3	9200	9303	103	14·6	6
1953	9085	9204	119	16·9	6	9204	9316	112	15·9	7·5

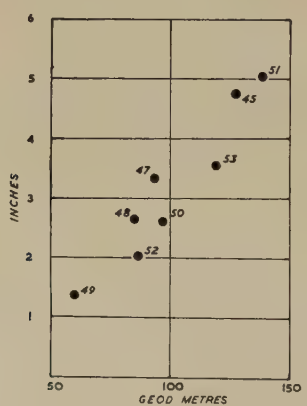

Figure 5. Winter-rainfall at Hay plotted against the geodynamic-height difference of 300-mb surface over Rathmines and Laverton.

6. PRECIPITATION AND THE 300-mb ZONAL WIND

Fig. 5 shows a plot of winter rainfall at Hay against $\Delta\phi$, the geodynamic-height difference between Rathmines and Laverton. The existence of a relation appears obvious in this case. For other stations the relation has not always the same linear character, but there is a strong tendency everywhere in south-west New South Wales for heavy winter rains to be associated with strong upper winds. The opposite holds in summer. This is shown readily by a Rank Correlation.

The lowest seasonal rainfall for a given station is ranked as Number 1, the second lowest as number 2 and so up to the highest. The ranks so obtained are related to the

ranking numbers for the upper wind by Spearman's Ranking Correlation Coefficient,

$$r_R = 1 - \frac{6\Sigma d^2}{N^3 - N}$$

where Σd^2 is the sum of the squares of the difference between upper wind and rainfall ranks, for each year, and N is the number of ranked seasons, viz. 8 for winter, 6 for summer and 9 for autumn and spring in the present case. The standard error of a Rank Correlation, approximately,

$$\sigma_R = (N - 1)^{-\frac{1}{2}},$$

is shown in Table 4 for each season together with Ranking correlation coefficients for a number of stations.

TABLE 4. RANKING CORRELATION COEFFICIENT OF MEAN ZONAL WINDS AT 300 mb AND OF RAINFALL IN SOUTHERN N.S.W. (1945-1953)

	Station	Ranking Correlation Coefficients			
		Winter	Summer	Spring	Autumn
	Wentworth	0·83	− 0·78	0·72	− 0·11
	Euston	0·96	− 0·94	0·28	0·05
Western	Balranald	0·90	− 0·60	0·55	0·18
plains	Hay	0·88	− 0·26	0·61	− 0·07
	Deniliquin	0·81	− 0·89	0·50	0·40
	Yanko	0·83	− 0·60	0·63	0·26
	Narrandera	0·76	− 0·89	0·90	0·13
Western	Wagga	0·74	− 0·14	0·92	0·20
slopes	Albury	0·76	− 0·20	0·83	0·60
	Tumbarumba	0·72	0·31	0·87	0·20
	Adaminaby	0·12	− 0·89	0·72	0·32
Eastern	Cooma	− 0·17	− 0·66	0·35	0·22
slopes	Bega	0·17	− 0·83	0·68	0·25
and coast	Kurrajong	− 0·24	− 0·94	0·45	0·28
	Sydney	− 0·21	− 0·71	0·46	0·47
	Standard Error	0·38	0·45	0·33	0·33

(a) *Winter*

It will be seen that the winter Ranking Correlation Coefficient for stations in the SW. plains is everywhere more than twice its standard deviation. For stations on the slopes it is somewhat smaller but still significantly large. On the eastern side of the mountains, on the other hand, the correlation for winter breaks down or is possibly negative. This is interpreted readily as a result of the Föhn effect mentioned at the end of Section 5.

(b) *Summer*

Outbreaks of cold polar maritime air – if they do reach southern New South Wales in summer – are usually shallow. Fronts are often dry without any accompanying rain belt. The summer development of rain from big cumuli will be hampered by strong winds aloft and the resulting vertical shear. A strong upper current from the west would presumably counteract also the monsoonal inflow of moist warm air from the Tasman and Coral Sea to the east. A negative correlation between the upper westerlies and rainfall both inland and on the coast may hence be expected. This appears to be in fact confirmed by Table 4.

With 6 ranks the standard deviation of this correlation $\sigma_R = 0{\cdot}45$. The predominantly negative character of the correlation is immediately evident. It is of interest to note that the negative correlation is weakest on the western slopes where it might be expected that the westerlies would produce some orographic rain even in summer. This may be the explanation for the insignificance of the negative correlation for Wagga and Albury. Tumbarumba in a valley at the western foot of the Snowy Mountains is the only station to show a small positive correlation.

The negative correlation between upper westerlies and rainfall should be particularly pronounced on the eastern slopes, as is indeed the case. The smaller magnitude of the coefficient for Cooma as compared with Kurrajong Heights – at the same altitude on the eastern slope of the coastal range near Sydney – is due to the situation of the former station in an inter-mountain valley, shielded not only from the west but also from the east.

(c) *Spring and autumn*

With a positive correlation for winter and a negative one for summer, spring and autumn should have the character of transition seasons. In fact, the correlation is still significantly positive in spring. This is the case particularly on the western slopes, though as rain crosses the range more readily in this season than in winter, a positive correlation exists also for the eastern side. The significant correlation between spring rainfall and upper winds is probably due to the fact that the east-coast spring cyclones tend to draw some of their energy from the westerly circulation.

The autumn cyclones tend to have a more tropical character, particularly in March and April; the correlation between autumn rains and upper winds, though again predominantly positive, is not significant.

7. The interpretation of the climatic change in terms of atmospheric motion

The analysis of rainfall records shows a tendency for the average summer rains in southern New South Wales to increase from a minimum in the eighteen-nineties. The winter rains at the same time have tended to decrease from a maximum which occurred soon after the summer minimum. From the correlations in Section 5 and 6 this would appear to be in agreement with the assumption of a gradual decrease of the upper westerlies, both in summer and in winter, from a maximum at about the turn of the century.

The great decline of the spring and autumn rains presents a complication. Table 4 showed a significant positive correlation between spring rains and the strength of the upper westerlies. The sudden decline of the spring and autumn rainfall after 1894 would hence hardly seem to support the assumption of an all-year-round maximum of westerly flow during that period.

It is of interest in this connection, to examine the changing yearly distribution of the rainfall for an old-established station like Deniliquin.

TABLE 5. Thirty-year mean monthly rainfall values for Deniliquin (inches)

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1864-1893	1·04	1·17	1·48	1·70	1·83	1·66	1·27	1·48	1·78	1·76	1·27	0·66
1894-1923	0·91	0·94	1·21	0·92	1·50	2·17	1·40	1·40	1·50	1·32	0·90	1·11
1924-1953	0·91	1·22	1·13	1·37	1·30	1·37	1·34	1·38	1·01	1·71	1·00	1·09

The rainfall regime of the second period, with its sharp maximum in June is typical for the climate of South Australia, and would associate Deniliquin with a climatic region which normally is further west and south-west. On the other hand the heavy rainfalls of March, April and May, during the period 1864-1893 are more typical of the climate of eastern and north-eastern New South Wales; which means that during this period the climate of Deniliquin was more of the east-coast type. An eastward or north-eastward displacement of the climatic zone boundaries and a narrowing of the east-coast climatic region would not appear to be inconsistent with the assumption of a stronger westerly circulation.

The decrease in spring and autumn rainfall at the end of the last century was almost certainly due to the temporary absence of east-coast cyclones. In fact, isopleths of the percentual value of the decrease have very much the same shape as isohyets of a depression centred over eastern New South Wales. These east-coast cyclones derive some of their energy from the westerlies – hence the significant positive correlation in Table 4. Their development depends however also upon a supply of moist maritime air, from the Tasman Sea, in the lower layers and upon vertical instability. It is these last conditions which were probably absent during the period of dry springs and autumns after 1893.

The same happened in some recent years. Very stable conditions persisted for example for considerable intervals both in spring and in autumn, 1951. From Table 4 it may be seen that both these seasons tended also to deviate on the dry side from the ranking correlation. Both seasons were characterized furthermore by the development of very strong west winds, of jet-stream character, to the north over the eastern tropical and sub-tropical part of the continent. This is evidenced to some extent by Table 6 which gives the mean geo-potential height of the 300-mb surface over the two east-coast stations of Townsville (19° 15′S, 146° 42′E) and Amberley (27° 38′S, 152° 43′E) and the associated mean wind.

TABLE 6. Geo-potential height (ϕ) of the 300-mb surface over Townsville and Amberley and associated mean geostropic wind ($\overline{U}$)

	Autumn			Spring		
	Townsville geod. m	Amberley geod. m	$\overline{U}$ m/sec	Townsville geod. m	Amberley geod. m	$\overline{U}$ m/sec
1949	—	9462	—	9562	9419	15·3
1950	9617	9478	13·9	9589	9433	16·7
1951	9626	9465	17·3	9613	9419	20·4
1952	9641	9493	15·8	9637	9463	18·6
1953	9652	9520	14·1	9624	9464	17·1

Comparison with Table 3 shows that the upper west winds over the tropics are frequently stronger than those over the southern part of the continent. This is typical for the Australian east coast. Turbulence will tend to produce a polewards transport of mass across the contour lines and, hence, upper convergence and subsidence on the southern, poleward side of the belt of maximum wind-strength. In fact it is this meridonal circulation which is probably the primary phenomenon. The data suggest that it was developed comparatively strongly in 1951.

The dry conditions in the spring and autumn of that year were not limited to New South Wales. The very strong upper winds over tropical Queensland were associated with a very pronounced vertical wind shear. Such a shear inhibits the full development of monsoonal cumulus clouds and it can only arise in the first instance if a stably stratified atmosphere opposes vigorous vertical mixing (Rossby 1951). In fact, the monsoon did fail partially and the year 1951 is known as a severe drought-year with heavy stock losses in the eastern tropical parts of the continent.

The same combination of dry springs and autumns in the south-east, with a monsoon failure or exceptionally short monsoon seasons in the tropics, occurred in 1922-23, in 1933-35 and it appears to develop again in 1954. The existence of a connection becomes still more probable when it is observed that the long-term fluctuations of the monsoon seem to parallel the changes that occurred in the spring and autumn rains and to a considerably lesser degree in the summer rains of the south. Table 7 gives mean yearly rainfalls for 3 stations which depend for their water almost exclusively on the wet summer-monsoon season.

TABLE 7. Average yearly rainfalls in inches for stations in tropical Queensland

Period	Georgetown	Townsville	Gilbert River
1872-1896	37·48	53·97	38·32
1911-1940	28·76	39·69	26·93

The heavy decline of the mean monsoon rainfall from the first period to the second one matches the information summarized in Table 1.

In conclusion it is assumed that a strong equatorial development of the upper westerlies weakens or shortens the monsoon in the Australian tropics and at the same time inhibits the development of wet east-coast cyclones in the south. Such a development was apparently rare between the eighteen-sixties and the eighteen-nineties – a period of high average monsoon-rain in the north, wet springs and autumns in the south and extension of a coastal type of climate comparatively deep in to inland New South Wales.

In the middle of the eighteen-nineties the mean strength of the westerlies seems to have increased rather suddenly and the mean position of the maximum shifted towards the equator. This was associated with repeated monsoon failures or short monsoon-seasons in the north. In southern New South Wales and eastern Victoria this period was characterized by an almost catastropic reduction of the spring and autumn rains, by comparatively dry summers and by fairly wet winters. A South Australian (Mediterranean) type of climate extended eastwards into south-western New South Wales. Since then, over the past forty years, the equatorial development of the westerlies appears to have become again a less frequent occurrence. Summers and, to a lesser degree, springs and autumns have tended to become wetter again and winter rains are becoming less abundant. These long-term fluctuations must have been associated with corresponding changes in the strength and pattern of the meridional transfer of heat and momentum between the tropics and temperate latitudes.

This interpretation of the climatic changes could probably be refined and tested by a study of the physical interaction between the monsoon and the westerlies, the effect of this interaction upon the development of east-coast cyclones, and by a more detailed statistical investigation of monsoon rainfalls and their relation to the temperate rainfall-regime.

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Provisional maps of the reliability of annual rainfall in East Africa

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Summary

East Africa is a dry country. Its increasing population, which is dependent on agriculture, seeks expansion into new agricultural lands. Most of the available land lies in regions of low and variable rainfall. One of the most important requirements in the selection of sites for future development is that they should be subject to a reasonably reliable rainfall suited to the potential crop.

Earlier investigations have shown that the distribution of annual and seasonal rainfall in East Africa can be assumed to be *normal* for all practical purposes. On this basis it is possible to calculate the probabilities of receiving certain selected minimum values suited to a crop, and these can be combined for mapping purposes. Maps of the reliability of annual rainfall at two selected levels illustrate the potentialities of the method. The meteorological background to the maps and their application to agricultural development are briefly discussed.

1. Average rainfall and agricultural expansion

Somewhat surprisingly, British East Africa (i.e., Kenya, Uganda and Tanganyika) is in the main a dry region, although it is subject to two monsoons, which usually follow ocean courses and bring moisture-laden air inland from the sea. Their failure to deposit water seems to lie in their being persistently divergent over large land areas, and in this they markedly differ from the convergent monsoons of southern Asia. Most of the rain over East Africa is associated with the movement of the inter-tropical convergence zone at the change of the monsoon. This zone separates the air masses originating in the northern and southern hemispheres, and the rain associated with it is very variable in intensity and amount, and is often inadequate for agricultural purposes. Fortunately, in the western regions of the territories, the inadequacy of rain is mitigated by another convergence zone resulting from the inflow of 'westerlies' from the Congo. These winds have their origin in the South Atlantic and they appear to lose little of their maritime nature during their passage over the low-altitude Congo forests. As a result, their convergence with the monsoon-air currents which have traversed the eastern parts of the continent produces a considerable amount of additional rain in those parts affected by it. This second convergence zone is virtually permanent in character, although its intensity and location vary considerably. Much of the year it lies somewhat to the west of the 30th meridian, but from time to time it advances into East Africa to the 33rd meridian and sometimes in southern Tanganyika as far as the 38th.

These sources of variable rainfall exert a profound influence on the life of the indigenous peoples. Most of the population is at present concentrated in the higher rainfall areas where bulk water and a crop of sorts can always be obtained. The population however, is increasing rapidly, and pressure on the land in these areas has reached a point where some people live little above subsistence level, so that there is an increasing demand for expansion into new land. Superficially, at least, such expansion seems possible in certain regions. For example, a map of the mean annual rainfall of East Africa (1953) shows that some large underpopulated regions of Tanganyika have an average of 30-40 in. of rain a year. As much of this territory has a single well-defined rainy season (November-May) and the rest of the year is dry, the seasonal rainfall is very similar to the annual rainfall. These regions therefore appear to be suitable for farming based on the growth of grain sorghums which require only some 20 in. of rain

in the season. With an average of over 30 in., it should be possible to obtain an 'effective' rainfall of this amount, even though evaporation in this summer period is high. The three main factors which would reduce the apparent value of these areas are the presence of the tsetse fly, lack of bulk water, and poor soils. However, not all the area suffers from lack of water and poor soils, and the tsetse fly can be driven back, given money to clear and maintain the selected areas.

It would therefore appear that when cleared of tsetse such areas would be suitable for agricultural expansion. This conclusion, however, is based on a consideration of the average rainfall only, and an average is of very limited value, and may even be misleading when considering agricultural potentialities. A knowledge of the average rainfall in his district is not of much use to the farmer, who really wants to know what his chances are of getting sufficient rain for the adequate growth of his crop. If his crop requirement is but little below the average for the crop season it is of the utmost importance to him to know how often this will or will not be attained. Some simple measure of reliability of rainfall is therefore needed to judge the regime of an area and assist the planner of agricultural expansion.

2. Rainfall reliability

Of the various ways in which the reliability of rainfall can be expressed, the most practical, from the agricultural standpoint, should be based on how often a farmer can withstand crop failure and still maintain adequate living standards. Few can afford failure more than about one year in three. The measure of unreliability is therefore chosen as the 30 per cent probability (or more) of failure to achieve a selected minimum amount of rainfall in the growing season. This does not preclude the possibility that on occasions the rainfall will be just below the selected minimum and a crop of sorts can be obtained. The limit is therefore an extreme one. Conversely, reliable rainfall is measured as the 5 per cent probability (or less) of failure to achieve the selected minimum. Various intermediate degrees of reliability may be chosen for special purposes. The selected minimum amount of rainfall will, of course, depend on the type of crop which is grown.

In order to calculate the reliability of rainfall by means of probability it is necessary to determine the form of distribution of rainfall over a series of years or seasons. Fisher (1948) examined the records of annual rainfall over a period of 90 years at Rothamsted, and showed that the distribution was not significantly different from *normal*. Our investigations of annual and seasonal totals at a number of stations in East Africa (1953) have also shown no significant differences from the *normal* distribution, although our longest series of records was perforce shorter than his. The distributions were sufficiently close to *normal* to make calculations of probabilities based on the assumption of normality adequate for most practical purposes.

On this basis it was possible to examine the records of rainfall published by the East African Meteorological Department (1932-1950).

Details of the calculations have been given in earlier papers (*East African Agricultural Journal* 1953 and 1954). Briefly these are as follows :

(1) Calculate the average m, and standard deviation s.

(2) Using the selected minimum level of rainfall x, calculate the quantity : $d = \frac{m - x}{s}$.

(3) From any table of the probabilities of the *normal* distribution determine the probability corresponding to this difference d. Two actual examples may help to make this clearer. Station A has an annual average of 26·44 in., with a standard deviation of 5·32. Then if the selected minimum fall is 30 in.,

$$d = \frac{26{\cdot}44 - 30}{5{\cdot}32} = 0{\cdot}67$$

As 30 is higher than the average, this difference of 0·67 corresponds to a probability of 75 per cent for a fall less than 30. Out of 15 years recorded, 11 were below 30 in. as compared with an expected number of 11·25.

Station B has an average of 48·37 and a standard deviation of 15·04. In this case, $d = \frac{48{\cdot}37 - 30}{15{\cdot}04} = 1{\cdot}22$.

As 30 is lower than the average, this corresponds to a probability of about 11 per cent for a fall less than 30. In 19 recorded years, 2 were below 30 in. as compared with an expectation of 2·09 years. At this point it would not seem out of place to stress once again the very limited value of the average. Station C has an average of 44·33 in. – somewhat less than that of B – but the standard deviation is only 6·96, so that the resulting probability of getting less than 30 in. is only 2 per cent – a better proposition from the agricultural point of view than station B.

Since annual totals were immediately available, it was decided to prepare provisional maps of the reliability of annual rainfall over the whole of East Africa before starting the more important maps of the reliability of seasonal rainfall. For completeness, rainfall data for Ruanda-Urundi (1951) have been included.

The 20-in. and 30-in. minimum levels were selected as being of most interest agriculturally, because in East Africa annual totals such as these, if well distributed in the single or double rainy season seem adequate to permit the satisfactory growth of either sorghum (20 in.) or maize (30 in.). It is realized, of course, that maps of the reliability of seasonal totals and of reliability of certain selected patterns of rainfall within the season are of more importance to the farmer. These are being prepared, and will be issued later.

3. The maps

Data from some 1,200 recording stations scattered over Kenya, Uganda, Tanganyika, Ruanda-Urundi and the coastal islands were used in the preparation of the maps shown in Figs. 1 and 2. Most stations had about fifteen years' records, and all had at least ten, except for a few which were used in areas with no other suitable station in the vicinity. Fig. 3 shows the number of stations in each quarter of a degree square. It gives an indication of the relative accuracy with which the isopleths have been drawn in different areas. For example, the highland areas of Kenya are well covered by rainfall stations; the dry tsetse-infested regions of western Tanganyika are not.

The maps are closely hatched where the rainfall is reliable at or above the selected level and blank where the rainfall is unreliable. Intermediate hatchings show intermediate levels of reliability. The areas of high reliability are those in which the rainfall is unlikely to be less than the selected level more than once in 20 years over a long period (5 per cent or less probability). They are not areas of common average falls, for they include areas whose rainfall may be so high that even in a dry year it is well above the selected limit, and other areas of lower rainfall where the variation from year to year is so small that, although the mean is not much higher than the selected level, there is little chance of falling below it.

In a similar manner areas of low reliability are those in which the rainfall is *likely* to be less than the selected level at least 3 years in 10 over a long period (30 per cent probability or higher). Here too averages within the areas may vary widely, but all have a common likelihood of failure.

When the maps are compared, it can be seen that although some two-thirds of the land surface (including Ruanda-Urundi) reliably receives 20 in. of rain, more than two-thirds does not reliably receive 30 in. of rain a year. Under these conditions the expansion of agriculture will be difficult and emphasis must be placed on the use of grain crops, which are most efficient in their use of water, and on ranching.

The 20-in. map (Fig. 1) shows clearly the arid and semi-arid regions of central Tanganyika and northern and eastern Kenya. During both monsoons this huge area is

Figure 1.

one of virtually unceasing divergence. Local ' rain-shadows ' also occur, but they merely enhance a general desiccating influence. The remarkable feature of this map is the existence of fairly well-defined boundaries between the reliable rainfall zone of western Tanganyika and the central dry zone. Unlike the eastern side of the dry area, there is no well-defined change in topography, and the central plain continues almost to the

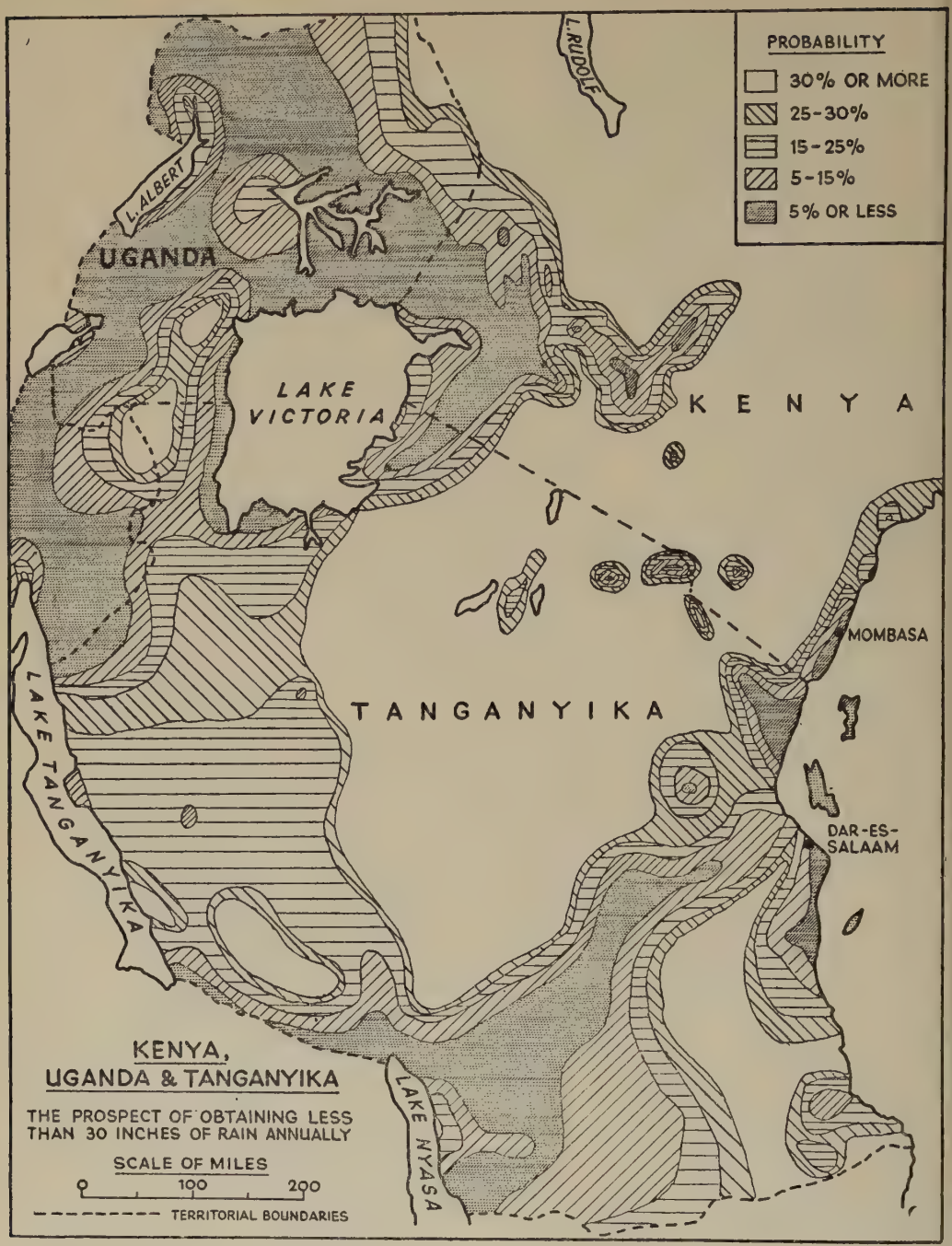

Figure 2.

shore of Lake Tanganyika. The incursion of the ' westerlies ' is responsible for these boundaries, since the eastwards advance of the associated convergence zone mitigates the desiccation accompanying the divergence of the monsoons. The eastern boundary of this reliable zone therefore approximates the average limit of incursion of the westerlies.

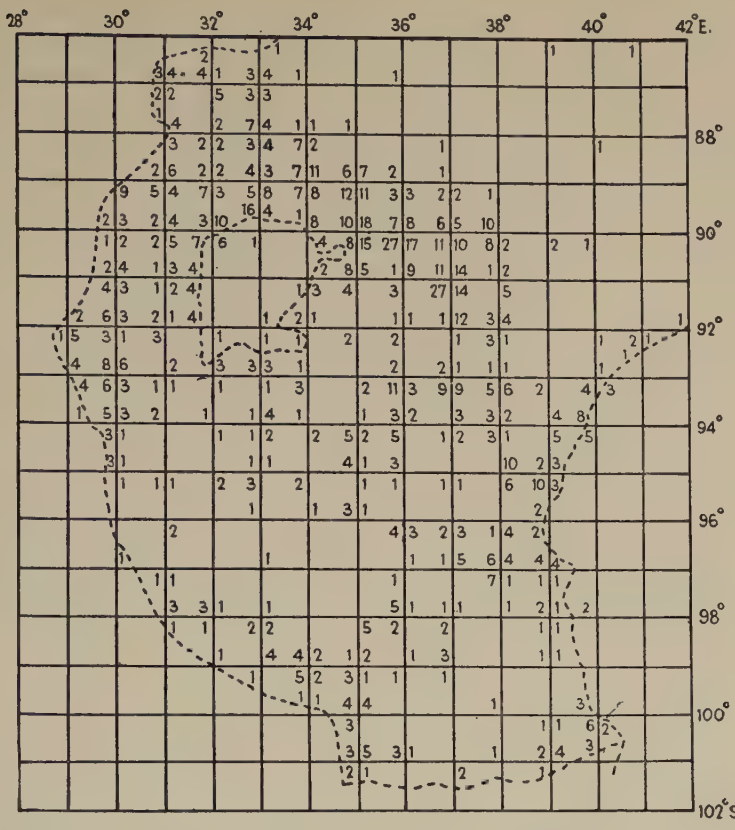

Figure 3. A map of East Africa showing the number of rainfall stations used for the preparation of the reliability maps in each quarter of a degree square.

The 30-in. map (Fig. 2) is probably more significant from a meteorological point of view, as dissimilar climatic regions are more clearly defined. The reliable-rainfall areas are, in the main, coincident with the mountainous regions where orographic effects account for much of the rain. The main exceptions are the elephant-grass regions of western and northern Lake Victoria, the riverine area of northern Uganda, and the Rufiji Delta.

The Lake Victoria region has its own peculiar wind circulation. The off-shore breeze at night tends to maintain storms which have formed in the eastern Lake before sunset, and which drift towards the west on the main current. The consequence is the well-known, relatively violent, morning storms which affect the shores west of the 33rd meridian.

The swamps and lakes of northern Uganda evaporate and transpire large quantities of water vapour into an atmosphere subject to a great deal of convection. This atmosphere is unstable throughout the SE. monsoon period, and the acquisition of the additional moisture may be effective in producing more rain.

The more reliable rainfall in the Rufiji Delta, and elsewhere along the coast, would appear to be due to the passage of relatively moist, cool, maritime monsoon air over the relatively hot swampy land; that is, much the same cause is operative as in northern Uganda, but the effects are not confined to the SE. monsoon period.

The dry areas of diverging monsoon are again clearly shown, and in addition, the downland area between the Katonga and Kagera Rivers in western Uganda is shown as

a significant zone of unreliable rainfall at this higher level. This zone is one of erratic unreliable rainfall, not primarily because of divergence, but because it is at most times of the year within the trajectory of air masses which have already lost their moisture orographically. Thus, during the SE. monsoon, and especially in July/August when the monsoon has tended to veer to southerly, west of the 32nd meridian, it is in the lee of the extensive high ground between the estuary of the Kagera River and northern Lake Tanganyika. This high ground filters off the moisture before it can reach the Ankole Downlands which are therefore in a rain-shadow. During the NE. monsoon the air mass crossing the zone is one which has travelled southwards from the Sahara or south-westwards from the Abyssinian massif. In either event, the air mass is dry as a consequence of the subsidence which accompanies the divergence of the monsoon. The bulk of the rainfall which occurs in this zone is attributable to the passage of the doldrum inter-tropical convergence zone in April, May and November, and to the temporary incursions of the Congo westerlies. Instability is general over wide areas when either of these factors determines the synoptic situation.

Again the westerlies are undoubtedly an effective agent in raising the probability of rainfall in western Tanganyika, as was discussed for the 20-in. map. Not infrequently the westerlies are evident as a wind drift above the monsoon; they still exert their influence, however, for even at heights of 10,000-12,000 ft above sea level they can promote instability in an air mass that is otherwise ineffective as a vehicle of rain production.

4. Some uses of the maps

Once the preliminary calculations of mean and standard deviation are completed, it is easily possible to determine the probability of receiving any selected amount of rainfall and to use this for mapping purposes. A number of detailed maps of this kind have been prepared for individuals and organizations. For example, a detailed map of the reliability of annual rainfall at the 30-in. level in the European areas of the Kenya Highlands was prepared for Troupe (1953). He showed in his ' Inquiry into the general economy of farming in the Highlands ' that areas which do not receive a reliable 30 in. of rain a year are devoted to cattle and their products, whereas areas which receive this amount reliably are given over to arable and plantation crops.

Although the maps were prepared in order to assist the agriculturist, their use is naturally not exclusive to this purpose. For example, if they are compared with Gillman's Vegetation Map of Tanganyika (1949), they show that his areas of *woodland* correspond to regions which reliably receive 20 in. of rain a year, whereas they do not resemble any areas in which the distribution is reliably at least 30 in. a year. This suggests that his *woodland* types can live on some 20 in. of rain a year, provided that they receive it regularly, and they do not need the extra 10 in. for survival.

In a like manner it is possible to show that certain ticks which are the vectors of East Coast Fever, a disease which attacks cattle, are mainly confined to regions whose annual rainfall is reliably above 30 in., and that exceptions are found only in swampy areas or along river banks where the ground water is sufficient to ensure their survival in the absence of sufficient rain.

The map (Fig. 3) which shows the number of stations in each quarter of a degree square which have been used in the preparation of the main maps shows clearly that the boundaries of the different regions can only be approximately correct. The accumulation of further records, both meteorological and ecological, will no doubt necessitate future modifications of these boundaries. It is thought, however, that the overall picture of reliability of rainfall at these selected levels is reasonably correct.

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Annual rainfall areas of southern England

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SUMMARY

This paper attempts to define areas throughout which the fluctuation of annual rainfall is relatively uniform. A general pattern is defined on the basis of graphs of ten-year running means, and a more detailed pattern is presented from a consideration of individual yearly values of exceptionally high and exceptionally low rainfall. A comparison of the results obtained by these two methods is then made to enable the relative validity of the several areas defined to be assessed.

1. INTRODUCTION

Although so many varied aspects of rainfall conditions have been considered for Britain, little attention has been paid to defining areas throughout which fluctuations in rainfall amount are relatively similar for all stations. It is felt that such an attempt may yield results which will be of help in any consideration of the numerous problems of water supply. One approach, covering the whole of the British Isles, has earlier been made on the basis of correlation coefficients (Glasspoole 1922, 1925), whilst, on a smaller scale, graphs drawn for five-year running means have also been used for this purpose (Boyd 1937). This present paper represents an attempt to delimit these areal units for annual conditions over southern England, for which purpose some 130 stations were considered for the period 1900-1939.

2. THE GENERAL PATTERN

As a first approach graphs were constructed for each station to show the changes in rainfall amount throughout the period. For this purpose running (or overlapping) means were used, adopting as most suitable a ten-year interval, as was earlier done by Glasspoole (1941). From a study of these graphs five areas have been defined, each possessing certain relatively distinctive features as regards the direction and degree of annual rainfall fluctuations over the period 1900-1939. These five areas, together with examples of characteristic graphs, are shown in Fig. 1. The basic curve for Areas I, II and III rises from a low initial value to a maximum in one of the decades 1909-18, 1910-19 or 1911-20, from which it descends to a central minimum in 1917-26, although low values occasionally also characterize the decade 1913-22 especially at more exposed stations. From here rainfall increases to another peak in either 1922-31 or 1923-32, followed by an overall decline towards the end of the period. Thus a general 'M' form of curve is characteristic of most of southern England, but distinctive areal variations may be discerned within this general framework. These variations are emphasized by a study of the statistical significance of the difference between any two points on these curves. From such calculations it was found that for any one ten-year average to differ significantly from another, differences of four and a half to five inches were necessary in the south-west and three to three and a half inches in the east. It is with these values in mind that the characteristics of the several areas are defined.

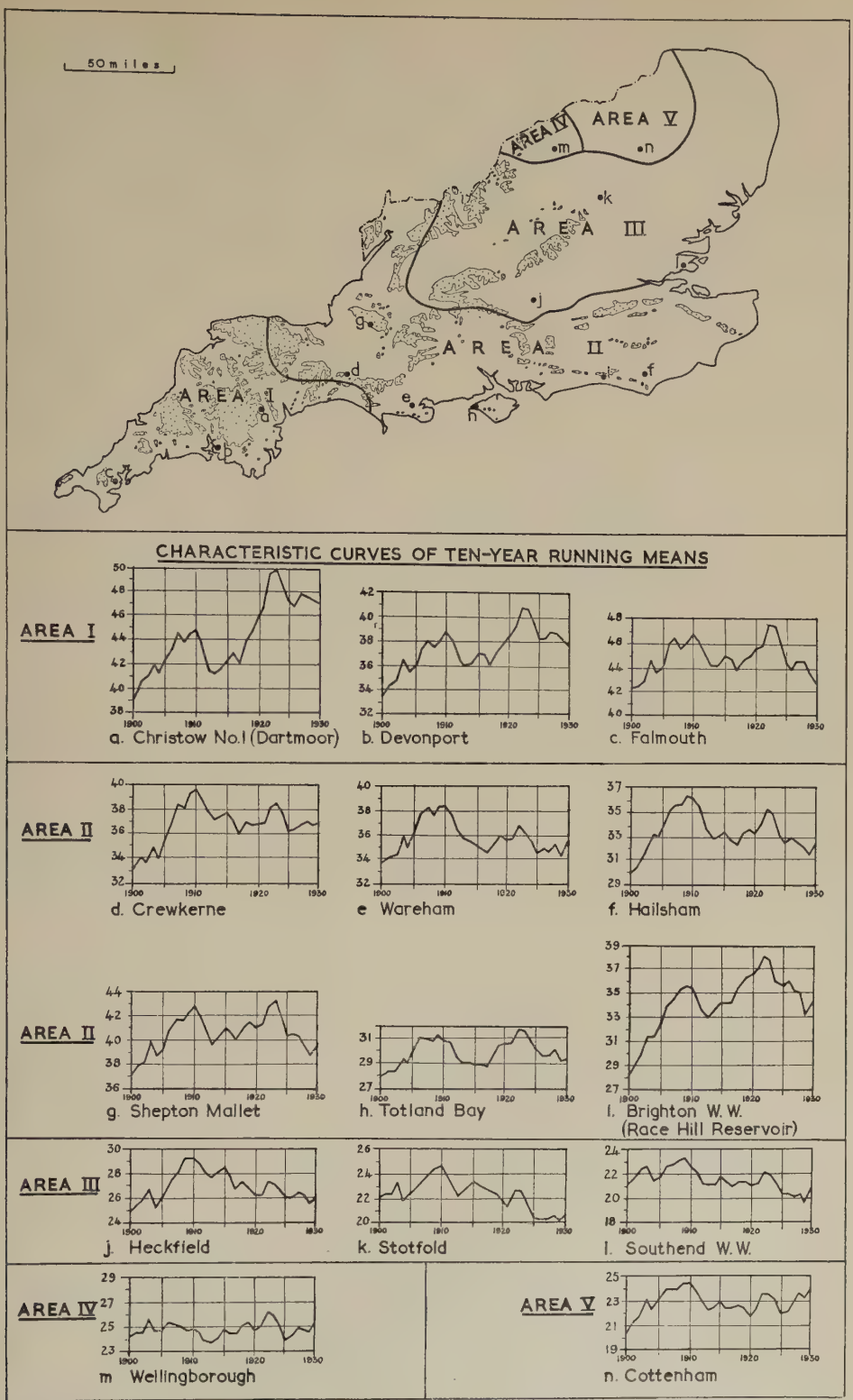

Figure 1. Annual rainfall areas of southern England based on ten-year running means, and examples of characteristic graphs. Data from *British Rainfall*. The years given beneath each graph are the *first* years of their respective decades. Rainfall scale in inches given on the left-hand side of each graph. Land above 500 ft O.D. stippled.

Area I. This comprises the south-west Peninsula plus the coastal belt of east Devon and west Dorset, throughout which both of the major increases in values (from 1900-09 to 1910-19, and from the central miminum to 1922-31) are significant features at most stations (Christow and Devonport, Fig. 1). Locally, however, these changes are not as great as four and a half inches (Falmouth, Fig. 1), but the graphs for such stations nevertheless display trends similar to, though less marked than, those of stations at which the differences are statistically significant.

A further feature of this area is the tendency for the second of the two major maxima to be the greater. This characteristic is most marked at stations exposed to westerly and south-westerly influences (Christow, Fig. 1), whilst in sheltered localities (Falmouth, Fig. 1) the difference between the two maxima is of less validity, although once again the graphs still display a similar trend.

Area II. This belt of country, from the lower Severn valley through the West Country and the area south of the Marlborough and North Downs, is essentially a transition belt. Over the lower areas, and those sheltered from westerly influences, only the first of the two major rises (1900-09 to 1910-19) is statistically sound. Moreover it is this earlier maximum of the two which is the greater, though not always strikingly so (Crewkerne, Wareham and Hailsham, Fig. 1). Conversely, in more exposed situations, especially at higher altitudes, the second maximum tends at least to equal, and occasionally to exceed, the earlier one (as in Area I), whilst the rise from the central minimum to this second maximum again approximates to a difference of statistical significance (Shepton Mallet, Totland Bay and Brighton Water Works, Fig. 1).

Area III. In this area, extending from the Cotswolds to the east coast and from the Marlborough Downs to the Norfolk coast, exposure to westerly influence is far less than in Areas I and II. This is reflected in the station graphs by the absolute dominance of the 1910-19 maximum which significantly exceeds the 1900-09 value at all save a few stations (Southend, Fig. 1), and by the irregular fall of the curve from this level to a minimum in 1929-38. The 1922-31 maximum here forms merely a halt or break in this gradual fall in values (Heckfield, Stotfold and Southend Water Works, Fig. 1). It is significant, especially from the point of view of London's water supply, that the whole of the Thames basin – apart from the valleys of the tributaries rising in the Weald – falls within this area.

Area IV. As one moves northwards towards the Fenlands and the east Midlands, the basic 'M' curve of Areas I-III begins to give way to new curve-forms. In Area IV (the east Midlands) fluctuations were only on a minor scale, and even the adoption of three inches as a criterion of significant difference cannot render the curve anything other than featureless (Wellingborough, Fig. 1).

Area V. In the Fenlands, on the other hand, the curve is in many respects comparable to that of Area III, being distinguished from it by the tendency for values to rise between 1920-29 and the end of the period (Cottenham, Fig. 1). In both of Areas IV and V, however, the relatively few stations involved render the clear definition of characteristics difficult.

This method suggests that conditions over most of southern England (Areas I-III) are relatively uniform, and also indicates the possible limits of these conditions in the Fenlands and east Midlands (Areas IV and V). Moreover, minor yet significant variations within this relatively uniform area are also stressed. A relationship between these and exposure to, or shelter from, influences from between west and south, is strongly suggested, both by differences between individual station curves (especially in Area II) and by the general coincidence of the area boundaries with the major west-facing and south-facing relief features (Fig. 1). It is also significant that there is a fairly close correspondence

between several of these areas and some of the rainfall provinces defined by Crowe (1940) on the basis of the seasonal distribution of rainfall. Thus Crowe's Province B includes most of Areas I and II, whilst Area III bears a close relationship to Crowe's Province G.

3. The detailed pattern

The curves used in this analysis are necessarily generalized in their construction, smoothing out the differences, both of magnitude and time of occurrence, between individual years of high and low rainfall. It is therefore both pertinent and necessary to consider the possibilities of analysing conditions on the basis of individual yearly values. This should help either to confirm or invalidate the areas defined in Fig. 1, and if it confirms them, it will also provide a more detailed and complete picture of their characteristics.

In a recent paper (Gregory 1953) annual rainfall conditions over southern England were classified as to the years of occurrence at each station of exceptionally wet and exceptionally dry values, and it is on the separate consideration of these two sets of conditions that the present areal delimitation is based. For this purpose, exceptional conditions were assumed to have occurred at a particular station if the precipitation for that year exceeded the upper quartile (exceptionally wet) or fell below the lower quartile (exceptionally dry) by more than ten per cent of the median.

The obvious method of grouping together those stations experiencing high or low rainfall (as defined above) in the same years is not a practical proposition, for the necessity of establishing a numerical dividing line between exceptional and non-exceptional conditions creates innumerable anomalies between adjacent stations. Some method was therefore required which would allow the inclusion with a group of stations with comparable records of neighbouring stations which only just failed to receive exceptional rainfall in one or more years. Only by such a process could the grouping of stations with basically similar records be achieved. As strictly statistical methods do not allow for even such a minor, and necessary, intrusion of subjective assessment, a graphical method was evolved by which these minor adjustments could be made.

(*a*) *Regions*

To illustrate the method whereby the regions are defined, an example based on exceptionally wet conditions may be used. A diagram (Fig. 2) was constructed for each station to indicate years of exceptionally high rainfall (a black square) and years with rainfall higher than normal but not exceptionally so (diagonal shading), these being those years in which rainfall exceeded the upper quartile (the upper limit of the normal) by

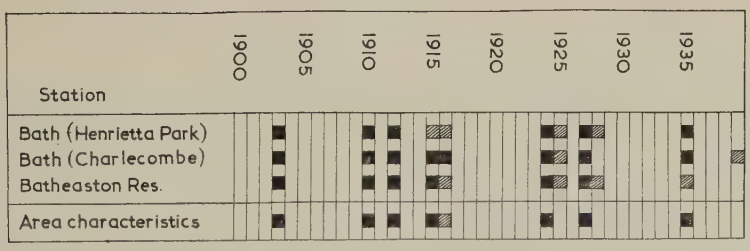

Figure 2. Method of regional delimitation. Data from *British Rainfall*, 1900-1939. (a) station characteristics: black square = exceptionally wet year; diagonal shading = year with transitional conditions. (b) area characteristics: black square = half or more stations exceptionally wet and the rest transitional; diagonal shading = less than half the stations exceptionally wet and the rest transitional.

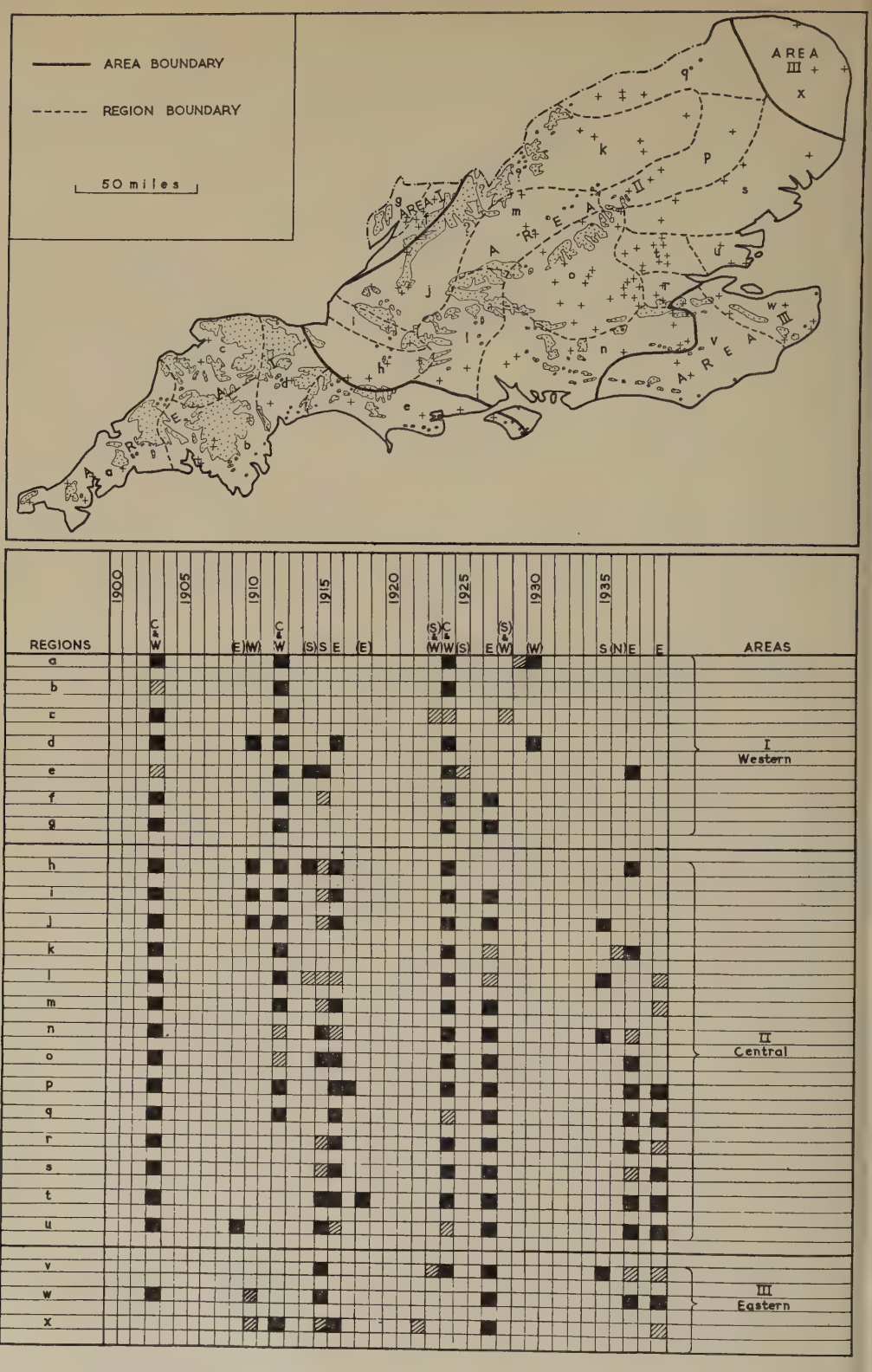

Figure. 3. Annual rainfall regions and areas of southern England based on exceptionally high rainfall conditions, and diagram of regional characteristics. Data from *British Rainfall*, 1900-1939. (a) Stations possessing all the regional characteristics are shown by a cross; those with slight discrepancies are shown by a black dot. (b) black square = half or more stations exceptionally wet and the rest transitional. (c) diagonal shading = less than half the stations exceptionally wet and the rest transitional. (d) letters at head of yearly columns give classification of distribution from Gregory (1953), brackets indicating localised distributions. (e) land above 500 ft O.D. stippled.

less than ten per cent of the median. All years with normal or lower than normal rainfall were left blank. Visual comparison of the records of three neighbouring stations near Bath (Fig. 2) suggests that slight differences exist even in such a small area, as has been shown in detail earlier (Balchin and Pye 1948). This serves to emphasize the impossibility of regional definition by a simple grouping of stations which had exceptional conditions in comparable years. Therefore, to facilitate the grouping of those stations with records sufficiently similar to justify the resulting regions being described as homogeneous, all those years characterized by diagonal shading in Fig. 2 were considered as transitional. Thus, if for any one year certain stations out of a group of stations record exceptional values and the rest record transitional conditions in the above sense, then the record for that year is taken as homogeneous for the whole group of stations. Conversely, transitional values recorded from an area where no exceptional values are to be found are taken as merely local minor increases in an area of basically normal rainfall.

Thus, in the case of the stations in Fig. 2, all three experienced exceptionally wet conditions in 1903, 1910, 1912, 1924 and 1927, this being indicated in the fourth line of the diagram, which presents the area characteristics, by a black square. In 1915 and 1935 two (i.e., in a larger sample, over half) of the stations had exceptional conditions and the remainder transitional conditions, and this is also summarized in the fourth line of the diagram by a black square. In 1916, on the other hand, one (i.e., in a larger sample, less than half) of the stations had exceptional conditions, but as the rest were transitional in nature, this is still a homogeneous record within the above definition. To indicate its lesser degree of development, however, the area characteristics are depicted by diagonal shading. Finally, although all or some of the stations experienced transitional conditions in 1925, 1928 and 1939, the absence of exceptional records means that these were not years of high rainfall in the Bath area. Although in theory this method appears highly subjective, in practice there is rarely any possibility of a difference of opinion upon the affinities of the record of any particular station. The few stations for which two alternatives were possible usually lay near the boundary of the region (Figs. 3 and 4).

The characteristics of regions arrived at by this method require strict definition. Thus a ' high-rainfall region ' is an area within which ' if some stations experienced exceptionally wet conditions in any one year, then all other stations experienced conditions at least wetter than normal,' whilst a comparable definition may be made for ' low-rainfall regions.' The many small, individual, homogeneous regions thus arrived at, together with their characteristics, can be seen in Figs. 3 and 4.

(*b*) *Areas*

The large number and small size of these regions creates a rather confusing picture, whilst the distinction between one region and another is often based on a difference of only one year, as Figs. 3 and 4 show. A method of grouping regions of relatively similar characteristics into larger areas was therefore evolved, based on the recent classification of annual rainfall-distribution types over southern England already mentioned (Gregory 1953). In that paper was given the relative location over southern England of exceptionally wet and exceptionally dry annual rainfall conditions for each year of the period 1900-1939, the category into which each year falls being indicated at the head of the diagrams in Figs. 3 and 4. A distinction is made between major distributions and those localised ones which affected only one or two regions (given in brackets); it is the former of these which provide the basis for grouping regions into larger areas.

In the majority of cases the distributions classified as central affected all or nearly all of southern England, and were thus of little use for areal differentiation. Again, the northern

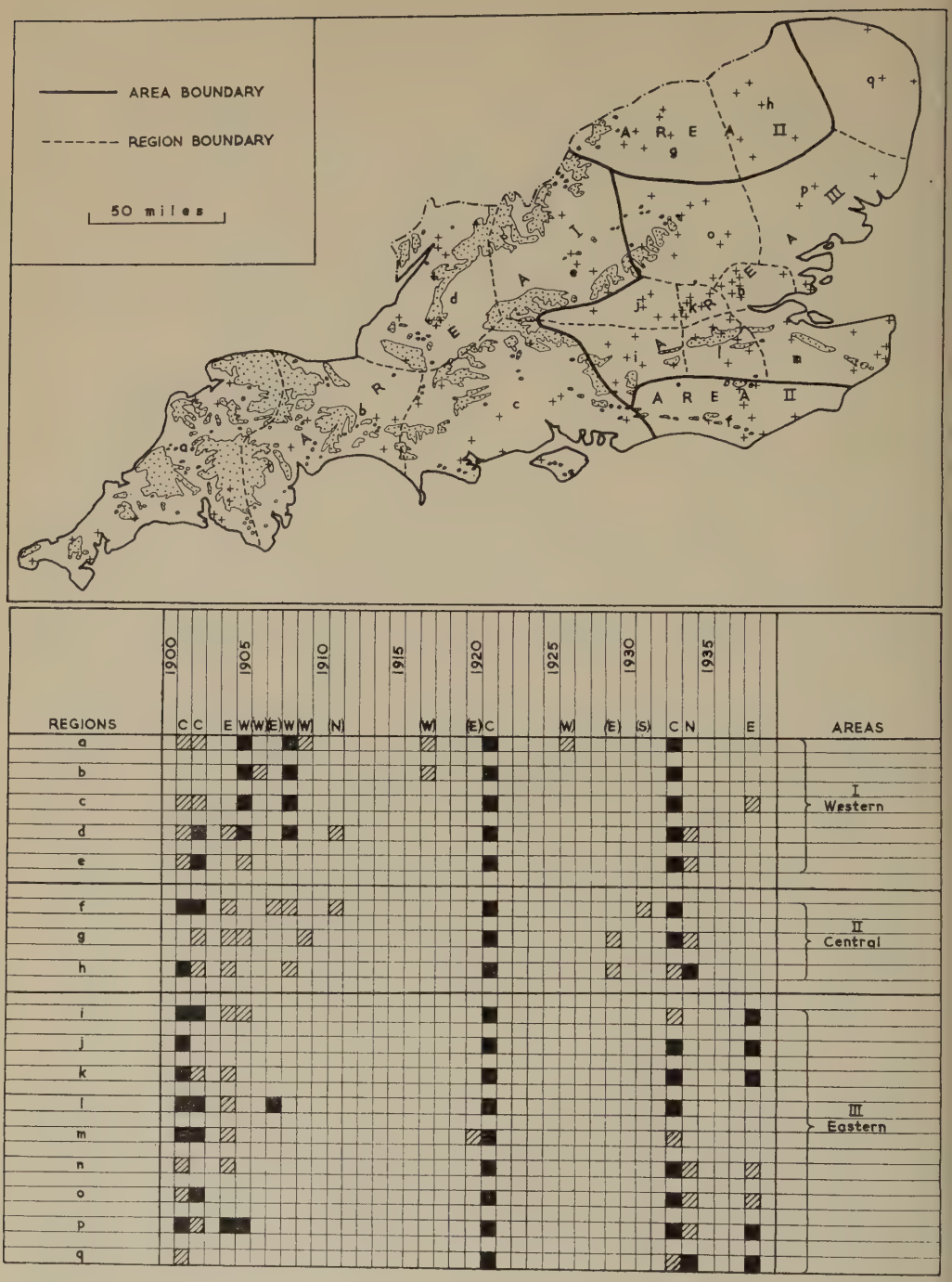

Figure 4. Annual rainfall regions and areas of southern England based on exceptionally low rainfall conditions, and diagram of regional characteristics. Data from *British Rainfall*, 1900-1939. (a) Stations possessing all the regional characteristics are shown by a cross; those with slight discrepancies are shown by a black dot. (b) black square = half or more stations exceptionally dry and the rest transitional. (c) diagonal shading = less than half the stations exceptionally dry and the rest transitional. (d) letters at head of yearly columns give classification of distribution from Gregory (1953), brackets indicating localised distributions. (e) land above 500 ft O.D. stippled.

and southern distributions were so few as to be of relatively little value in this connection either. As a result, any differentiation had to be based on the occurrence or non-occurrence in each region of the major eastern and western distributions. A simple three-fold distinction was made into (i) those regions which were affected by at least half of the major western distributions but by less than half of the major eastern ones, these being grouped as Area I (Western); (ii) those regions which were affected by relatively similar proportions of the major eastern and western distributions i.e. either over half or less than half of both categories, these being grouped as Area II (Central); (iii) those regions which were affected by at least half of the major eastern distributions but by less than half of the major western ones, these being grouped as Area III (Eastern).

In the case of exceptionally high rainfall conditions the definition of these areas was based on four eastern distributions and three western-and-central ones; for exceptionally low rainfall conditions there were only two eastern and two western distributions, as there were fewer years of exceptionally dry conditions and many of these affected all or most of southern England. The resulting areas may be seen in Figs. 3 and 4, but it should be remembered that frequently almost the whole of southern England acts as a single unit, as is indicated by the large number of central distributions included in the classification.

4. The overall pattern

The relationship between the areas based on high and low rainfall considered separately (Figs. 3 and 4) and those based on general conditions as represented by ten-year running means (Fig. 1) can readily be assessed. The close similarity between Area I in Fig. 3 and Area I in Fig. 1, and between the northern section of Area II in Fig. 4 and Areas IV and V in Fig. 1, is clearly apparent, but there are equally many areas where such

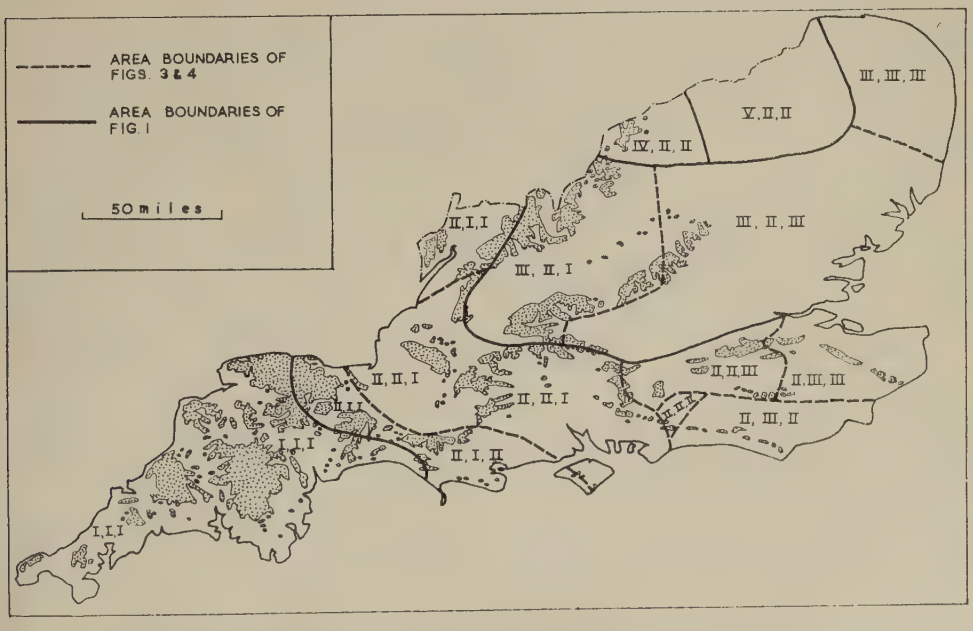

Figure 5. Annual rainfall areas of southern England based on yearly values (Figs. 3-5) and on ten-year running means (Fig. 1). Data from *British Rainfall*, 1900-1939. (a) Roman numerals present the areal classifications of the three preceding maps, in the following order – Figs. 1, 3, and 4. (b) Land above 500 feet O.D. stippled.

relationships are not so obvious. A more valid comparison, however, would be between Fig. 1 and a map combining all the areas shown in Figs. 3 and 4, this composite picture being presented in Fig. 5.

This latter map emphasizes especially the essential unity of the south-west Peninsula, and also the validity of the change in conditions as the Fenlands and east Midlands are approached. Moreover, the larger part of the extensive Area III in Fig. 1 is again seen in Fig. 5 to be relatively homogeneous, although with some changes from south-west to north-east. Finally, both the transitional nature of Area II in Fig. 1, and the differences within it, are stressed by the complex pattern of areas based on exceptional conditions which it includes.

5. Conclusions

It would seem that by the use of these methods it is possible to delimit, in general terms, at least the more significant of the areas over which annual rainfall fluctuations are relatively uniform. A definition of such areas on the basis of ten-year running means provides a working foundation; the analysis of exceptionally wet and exceptionally dry conditions separately helps to indicate both those areas of greatest uniformity, e.g., Areas I and III in Fig. 1, and those within which there are considerable local differences, e.g., Area II in Fig. 1. Moreover, both methods emphasize the considerable degree of uniformity which exists over most of southern England, as that term is here used, and the relatively small number of years in which significant differences occur within the area. It is hoped that the results achieved by these methods will prove of value especially in the study of the many complex problems of water supply.

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The distribution of wave heights and periods in ocean waves

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(Manuscript received 5 March 1954)

Summary

Automatic records of deep ocean waves obtained by a British Ocean Weather Ship have been analysed. The results are compared with those of similar analyses made by other writers. Relations between the parameters describing wave heights are shown to be in close agreement with deductions made by Longuet-Higgins from a theoretical model, over a range of wave heights which includes all but the most exceptional conditions. Wave-period parameters are also calculated, but relationships between them are not so clear-cut as for wave heights.

1. Introduction

A number of investigators have analysed wave records to determine statistical relations between the various parameters describing a particular record. For example Putz (1952) analysed 25 records from underwater pressure recorders in various depths of water between 45 and 90 ft off the California coast, and Seiwell (1948) carried out a series of analyses of similar records from the North Atlantic; more recently Watters (1953) has analysed pressure records made by an Admiralty wave recorder at Greymouth in New Zealand. From a theoretical approach Longuet-Higgins (1952) deduced relations between the root-mean-square wave height, the mean height of the highest one-third of the waves and the most probable height of the largest wave in a given interval of time. His results are in good agreement with those published by the investigators mentioned above.

The National Institute of Oceanography has recently developed an instrument (Tucker 1952) which may be fitted to the hull of a ship below the water-line and which continuously records the water-level. A feature of the instrument is the elimination, by an ingenious device, of the ship's own vertical motion. When the ship is stopped the record may be expected to provide wave measurements more detailed and accurate than any obtained by eye estimation, no matter how experienced the observer. Such an automatic record can be made anywhere at sea. The instrument responds to waves with periods between 5 and 25 sec and within this range there is practically no attenuation of short-period waves such as is inherent in bottom pressure recorders.

One of the instruments has been installed in a British Ocean Weather Ship, and through the kindness of the Director of the Meteorological Office and the Director of the National Institute of Oceanography a series of records, made three-hourly in February and March 1953, have been made available for analysis. This paper summarizes the results. As far as the writer is aware these are the first instrumental records of deep ocean waves to be analysed.

2. Method of analysis

This was quite straightforward. The height H of each wave from crest to trough was measured to the nearest foot and the period T of each wave taken off the time scale on the record to the nearest second. The records include a few taken while the ship was moving slowly to regain station; parameters for these have been calculated but none of them has been plotted on the diagrams, nor have they been used when calculating 'best-fit' straight lines or correlation coefficients. They are included in the table of results for the sake of completeness.

3. The results

The analysed results are shown in Table 1, and plots showing the relation between various pairs of quantities have been drawn. 138 records were made while the ship was stopped. The parameters are defined as follows :

H_m = mean of all wave heights in a record.

$H_{\frac{1}{3}}$ = mean of highest third of the waves in a record (the ' significant ' height).

H_{max} = height of maximum wave in a record.

σ_H = standard deviation of the heights of waves in a record.

ρ_{xy} = coefficient of correlation between x and y.

Corresponding symbols with T instead of H have corresponding meanings; it should be noted that $T_{\frac{1}{3}}$ is the mean period of the highest third of the waves, i.e. $H_{\frac{1}{3}}$ and $T_{\frac{1}{3}}$ refer to the same selection of waves from a record.

4. Significant height and mean height

The values of H_m and $H_{\frac{1}{3}}$ are plotted in Fig. 1 (a). The best-fit straight line through the origin has the equation

$$H_{\frac{1}{3}} = 1{\cdot}603\, H_m$$

and

$$\rho_{H_m H_{\frac{1}{3}}} = 0{\cdot}986$$

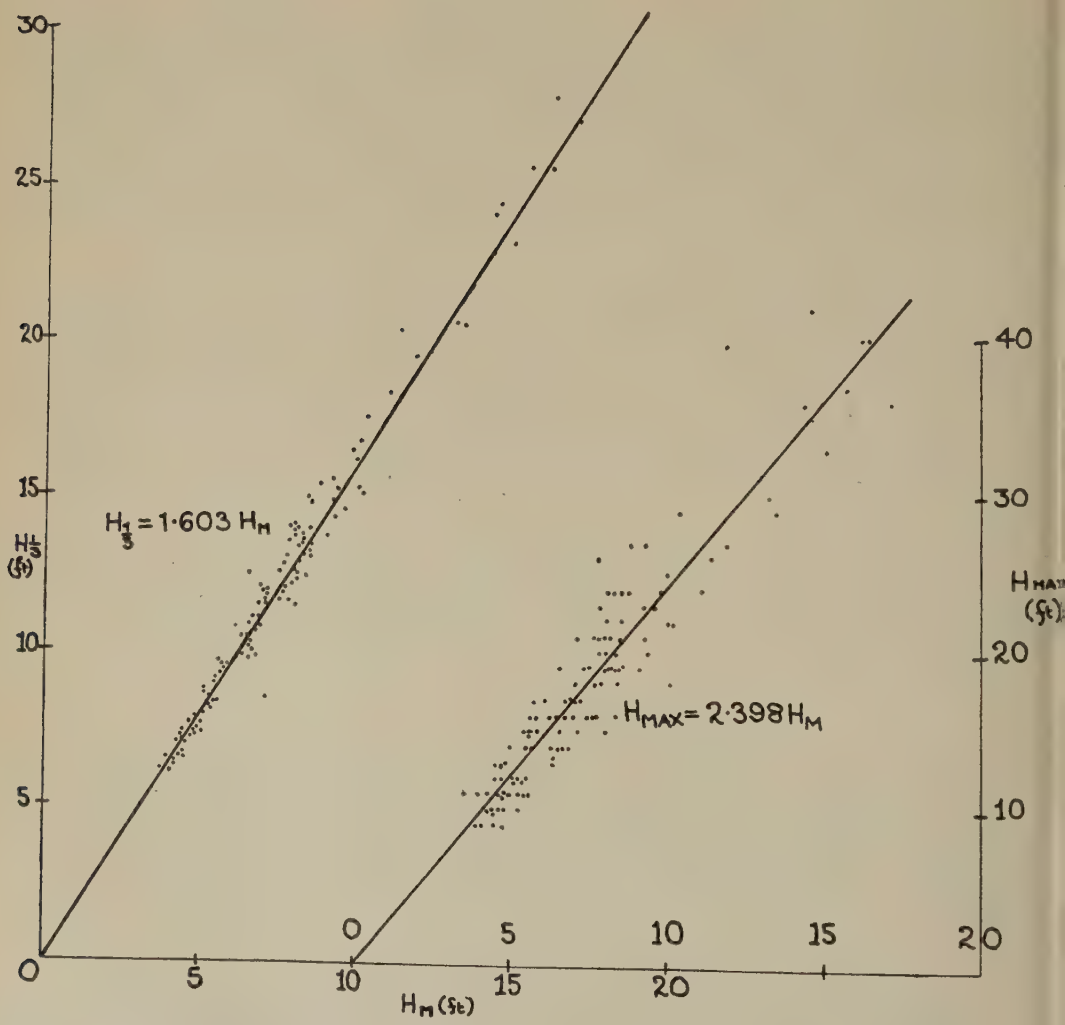

Figure 1. (a) Mean height of highest third of the waves against mean wave height. (b) Maximum wave height against mean wave height (abscissa shifted to the right).

TABLE 1. ANALYSES OF WAVE RECORDS (ship in position 53°N 20°W)

Date and time (GMT) 1953	Speed of O.W.S. (kt)	No. of waves	H_m (ft)	$H_{\frac{1}{3}}$ (ft)	H_{max}* (ft)	H_{max}/H_m	σ_H (ft)	T_m (sec)	$T_{\frac{1}{3}}$ (sec)	σ_T (sec)
22 Feb										
1145	–	56	8·57	13·08	21	2·45	3·94	9·23	9·67	2·32
1415	–	29	6·72	11·11	14	2·08	3·36	8·49	8·55	2·90
1745	–	60	8·06	13·80	20 (2)	2·48	4·94	8·03	9·10	2·51
2038	–	48	8·08	11·50	15	1·85	3·10	8·58	9·44	2·37
2335	–	66	8·06	12·57	19 (2)	2·35	3·81	9·29	10·33	2·20
23 Feb.										
0525	–	73	6·67	10·36	16	2·40	3·26	8·49	9·00	2·24
0830	–	50	6·86	11·06	14 (2)	2·04	3·51	8·56	9·60	1·86
1140	–	70	8·57	13·27	19	2·22	3·98	8·39	8·95	2·00
1500	2	78	8·95	13·84	22	2·46	4·82	7·78	9·20	2·23
1740	2	60	13·12	22·2	32	2·44	7·82	9·22	11·65	3·14
2035	2	68	13·60	22·43	33	2·43	7·65	9·16	11·22	3·21
2340	–	59	13·49	20·52	29	2·15	6·92	10·88	13·48	3·85
24 Feb.										
0245	–	26	15·07	23·22	33	2·18	8·32	9·58	12·11	3·59
0545	–	54	11·44	20·47	26	2·27	7·10	8·80	11·18	3·13
0845	–	52	14·63	24·47	42	2·88	8·72	9·02	11·23	3·18
1200	–	42	15·71	25·66	37	2·35	8·90	11·00	12·93	3·48
1440	–	66	16·37	27·91	40	2·44	9·00	11·23	13·82	4·06
1750	–	55	14·45	24·11	36	2·48	8·74	10·84	13·37	3·83
2030	–	51	17·18	27·23	36	2·09	8·41	12·29	15·23	4·27
2330	–	74	16·28	25·64	40	2·45	8·55	9·55	12·00	2·97
25 Feb.										
0235	–	62	11·17	18·33	24 (3)	2·15	6·50	10·38	12·76	3·63
0535	–	52	9·31	15·59	27	2·90	5·17	9·67	12·12	2·81
0845	–	65	9·33	14·32	21 (2)	2·25	4·21	9·63	11·45	3·21
1130	–	62	10·16	15·30	18	1·77	4·34	9·79	12·20	3·41
1435	–	77	11·95	19·52	27	2·26	6·34	9·20	11·28	3·13
1750	–	45	13·28	20·53	30	3·26	6·70	10·22	10·22	3·40
2035	–	67	9·30	14·95	23	2·47	4·71	9·75	10·67	2·85
2335	–	59	10·24	15·16	22	2·15	4·63	9·46	11·21	2·53
26 Feb.										
0240	–	53	9·91	16·67	24	2·47	5·79	9·97	12·17	2·63
0535	–	60	8·20	13·45	21	2·56	4·79	8·27	10·05	2·56
0900	–	61	10·41	17·57	29	2·78	6·38	9·29	10·91	2·76
1145	–	64	9·69	14·67	23	2·37	4·84	9·54	10·82	2·61
1445	–	79	10·16	16·81	22 (2)	2·16	6·09	8·38	9·81	2·28
1738	3	59	6·94	11·46	16	2·31	2·98	7·69	9·74	2·74
2100	3	74	7·97	12·48	20	2·56	4·09	9·25	10·84	2·37
2359	–	55	8·28	13·45	19	2·29	4·59	9·29	10·28	2·65
27 Feb.										
0245	–	64	5·98	9·30	15	2·50	2·91	9·09	10·50	1·61
0600	–	64	5·47	8·54	12 (3)	2·19	2·72	8·54	9·59	2·53
0835	–	93	4·49	7·23	10 (3)	2·22	2·36	7·92	9·20	2·57
1145	–	87	4·85	7·65	11 (2)	2·27	2·45	7·88	8·81	2·50
1440	–	70	5·11	8·13	12 (2)	2·35	2·67	8·44	10·31	2·93
1735	–	75	5·69	9·04	16	2·82	3·20	8·42	9·52	3·49
2100	–	56	7·59	11·73	16 (2)	2·11	3·82	9·48	11·42	3·05
2335	–	75	7·53	12·62	19 (2)	2·53	4·22	9·23	10·83	3·03
28 Feb.										
0230	–	83	8·17	14·00	24	2·87	5·06	9·15	12·04	3·63
0530	–	97	8·05	13·84	22	2·76	5·14	8·89	11·44	3·20
0835	–	66	7·73	12·05	16	2·07	3·67	9·60	11·38	3·07
1145	–	41	8·00	12·69	18	2·25	4·10	9·85	11·92	3·02
1430	–	48	6·91	12·19	17	2·51	4·40	8·27	10·63	3·08
1730	–	100	8·15	12·41	21	2·58	4·10	8·66	10·66	3·00
2035	–	67	7·71	12·00	16	2·07	3·59	8·19	9·33	2·74
2335	–	80	8·48	13·38	18	2·14	4·24	8·58	10·22	2·82

* Figures in parentheses indicate the number of occurrences of maximum.

Date and time (GMT) 1953	Speed of O.W.S. (kt)	No. of waves	H_m (ft)	$H_{\frac{1}{3}}$ (ft)	H_{max}* (ft)	H_{max}/H_m	σ_H (ft)	T_m (sec)	$T_{\frac{1}{3}}$ (sec)	σ_T (sec)
2 Mar.										
0245	–	66	4·51	6·75	9 (2)	2·00	2·10	8·56	8·30	2·20
0545	–	89	4·24	6·37	10	2·36	2·00	7·22	8·30	2·78
0835	–	70	5·07	7·48	11	2·14	2·11	8·07	8·73	2·09
1135	–	78	4·69	7·38	11	2·35	2·20	7·37	8·61	2·38
1435	–	55	5·33	8·26	11 (2)	2·05	2·60	9·05	10·42	2·92
1745	–	82	6·44	10·03	14 (3)	2·17	3·28	8·89	10·41	2·41
2030	3	124	6·25	9·79	13 (4)	2·08	3·09	7·85	8·92	2·85
2335	–	79	6·58	10·23	14	2·13	3·36	7·96	10·43	2·85
3 Mar.										
0245	–	76	5·82	9·88	16	2·75	3·59	7·48	10·92	3·21
0540	–	57	8·65	14·85	21 (2)	2·43	5·16	9·42	13·26	3·46
0830	–	78	6·64	12·52	19	2·86	4·12	8·51	11·24	2·92
1137	–	65	7·22	12·09	21	2·91	4·27	9·16	11·43	3·78
1435	–	69	7·81	13·64	18	2·31	4·75	9·05	11·00	3·22
1735	–	75	6·65	10·60	15	2·25	3·41	8·33	9·36	2·87
2035		69	7·88	11·73	16	2·03	3·52	8·80	9·87	2·90
2345	–	71	7·74	12·86	21	2·72	4·71	8·69	10·83	2·92
4 Mar.										
0245	–	54	6·61	10·61	14	2·12	3·45	8·43	9·55	2·73
0530	–	57	6·60	9·80	14	2·12	2·87	8·87	10·65	2·90
0840	–	70	6·76	10·71	16	2·37	3·47	8·26	9·71	2·85
1135	–	80	8·35	13·21	19	2·28	4·37	9·50	12·96	3·55
1450	–	70	8·09	14·12	19	2·35	5·30	8·49	10·71	3·70
1740	3	62	8·98	14·57	23	2·56	4·86	7·92	9·60	2·55
2045	3	71	7·90	11·92	20	2·53	3·83	7·80	9·60	2·69
2335	–	58	8·36	13·47	20	2·39	4·49	8·43	10·00	2·60
5 Mar.										
0235	–	80	8·35	13·70	24	2·87	4·76	8·26	10·26	2·67
0540	–	78	10·04	16·20	25	2·50	5·25	8·65	10·46	2·74
0845	–	48	7·81	13·06	26	3·33	5·06	8·00	9·80	2·88
1145	–	83	8·43	13·48	20 (2)	2·37	4·54	8·89	10·44	2·81
1435		74	9·12	13·78	19	2·08	4·09	8·83	9·61	2·54
1755		102	9·49	15·26	20	2·11	5·06	8·63	9·60	2·41
2040		90	8·57	14·00	24	2·80	4·68	8·42	9·76	2·35
2330	–	60	7·97	12·24	23	2·89	4·21	8·75	8·90	2·35
6 Mar.										
0235	–	77	6·38	9·87	13 (4)	2·04	3·07	8·15	9·74	2·81
0550	–	64	7·47	11·85	19	2·53	3·97	8·80	9·85	2·69
0835	–	71	6·73	10·08	16	2·39	3·11	8·30	8·80	2·37
1140	–	98	5·36	8·57	12 (2)	2·26	2·88	7·18	8·50	2·65
1445		104	5·11	8·06	12	2·35	2·63	7·79	8·81	2·24
1735		82	4·35	6·67	10 (2)	2·30	1·85	7·50	9·00	2·55
2035		54	4·50	7·44	10	2·22	2·28	8·35	9·56	2·66
2335	–	80	4·74	7·75	13 (2)	2·74	2·91	7·56	9·36	2·60
7 Mar.										
0235	–	87	5·06	7·75	12	2·37	2·46	8·90	10·84	3·16
0835	–	92	3·77	6·20	11	2·92	2·10	6·17	7·07	2·51
0540	–	62	5·08	8·24	14	2·76	2·79	7·31	9·19	2·63
1130	–	75	4·08	6·24	11	2·70	2·03	5·66	7·92	2·49
1440	–	72	4·16	6·39	9	2·76	1·64	6·94	9·56	2·80
1730	–	71	4·33	7·19	11	2·54	2·34	7·23	10·00	2·57
2035	–	59	6·35	10·10	16	2·52	3·35	9·01	10·15	3·15
2330	–	83	5·58	8·76	15	2·69	3·10	8·01	10·28	2·98
8 Mar.										
0235	4	76	5·41	8·70	15	2·78	2·97	8·54	10·85	3·27
0535	4	82	5·14	8·22	15	2·92	2·59	7·32	10·41	3·26
0835	–	52	4·91	7·72	10 (4)	2·04	2·41	8·31	9·94	2·95
1140	–	68	5·64	9·76	15	2·66	3·17	8·85	10·52	2·97
1437	–	61	5·39	8·47	10 (4)	1·85	2·53	7·70	9·84	2·81
1735	–	77	5·64	8·84	15	2·66	2·80	9·23	11·40	2·81
2030	–	68	5·16	8·81	12	2·33	2·97	8·53	10·71	3·26
2335	–	71	5·45	8·60	12 (2)	2·20	2·86	8·63	11·72	3·35

Date and time (GMT) 1953	Speed of O.W.S. (kt)	No. of waves	H_m (ft)	$H_{\frac{1}{3}}$ (ft)	H_{max}* (ft)	H_{max}/H_m	σ_H (ft)	T_m (sec)	$T_{\frac{1}{3}}$ (sec)	σ_T (sec)
9 Mar.										
0235	–	75	4·97	7·88	13	2·62	2·67	7·68	10·00	2·85
0545	–	59	5·74	9·60	15	2·61	3·50	7·24	9·70	2·93
0845	–	89	4·99	7·58	11	2·20	2·24	6·83	8·34	2·23
1130	–	121	4·67	7·32	13	2·78	2·48	7·32	8·82	2·47
1440	–	90	4·86	7·32	9 (7)	1·85	2·20	8·03	8·86	2·98
1740	–	91	4·48	7·31	10 (2)	2·23	2·32	7·54	8·62	2·45
2035	–	75	4·00	6·56	9 (4)	2·25	2·15	7·17	7·17	2·20
2345	–	89	4·52	7·19	12 (2)	2·65	2·40	6·92	8·29	2·26
10 Mar.										
0250	–	57	4·67	7·17	10 (2)	2·13	2·13	8·33	8·39	2·05
0540	–	74	4·75	7·33	12	2·55	2·37	8·39	9·77	2·51
0840	–	77	5·59	8·46	11 (3)	1·96	2·60	8·73	9·65	2·35
1140	–	58	5·74	9·29	14	2·46	2·93	8·96	10·11	2·47
1440	–	74	4·15	6·48	10	2·41	2·08	8·05	9·08	2·39
1740	–	53	5·61	8·94	11	1·96	2·74	9·26	10·75	2·51
2045		68	5·62	8·96	14 (3)	2·50	3·14	8·86	10·25	2·85
2330	–	64	4·75	7·45	10 (2)	2·08	2·03	8·11	10·10	2·85
11 Mar.										
0240	–	72	4·53	6·62	12	2·65	2·05	8·22	9·58	2·55
0535	–	66	4·86	7·36	11	2·26	2·35	8·04	8·82	2·53
0905	–	78	5·16	7·77	11 (3)	2·12	2·48	7·24	8·07	1·27
1145	–	78	5·66	9·08	14	2·48	2·92	7·30	8·16	2·21
1435	4	118	5·76	9·20	15	2·60	3·05	7·14	8·10	2·19
1730	4	77	5·40	8·20	12	2·68	2·68	7·57	9·57	2·68
2045	–	66	5·91	9·59	14	2·37	3·55	8·63	10·71	2·67
2340	–	55	6·45	10·53	13	2·00	3·46	8·92	12·26	3·48
12 Mar.										
0235	–	84	6·91	10·87	17 (2)	2·46	3·87	9·66	12·07	3·15
0530	–	85	7·90	14·10	21	2·66	4·98	9·33	12·35	4·37
0840	–	55	7·06	12·00	16	2·27	4·45	9·53	12·63	4·17
1130	–	49	7·14	11·75	17 (2)	2·38	4·24	9·63	13·50	4·21
1445	–	37	8·87	15·50	24	2·71	5·53	9·83	12·83	4·04
1735	–	58	8·49	15·00	19	2·24	5·45	9·83	13·95	4·37
2035	–	54	8·65	13·53	22	2·58	4·38	9·52	10·89	2·80
2335	–	59	7·24	11·84	15	2·07	3·83	8·92	12·00	3·24
13 Mar.										
0240	–	82	7·14	8·60	18 (2)	2·52	3·98	8·22	10·78	2·58
0540	–	65	7·87	12·44	20	2·54	4·52	8·31	10·67	3·03
0840	–	69	7·24	11·58	17	2·35	3·91	8·46	10·37	2·88
1150	–	53	6·89	11·56	14 (2)	2·03	4·01	8·24	9·84	2·63
1440	4	57	7·14	10·78	15	2·10	3·07	7·74	9·42	2·67
1740	4	78	7·37	11·88	19	2·58	3·74	7·65	10·50	2·95
2040	–	66	8·45	12·50	16	1·85	3·66	8·08	9·28	2·24
2330	–	74	8·19	12·87	19 (2)	2·34	4·53	8·01	9·25	2·41
14 Mar.										
0245	–	82	8·35	13·75	20	2·40	4·32	7·85	9·32	2·30
0545	–	76	6·63	10·88	16 (2)	2·42	3·78	8·55	9·08	3·10
0840	–	90	5·79	9·43	17	2·94	3·10	7·89	9·20	2·58
1130	–	73	6·19	10·76	17 (2)	2·75	3·55	8·08	9·28	2·47

The mean heights vary between 4 and 17 ft and the significant ($H_{\frac{1}{3}}$) heights between 6 and 28 ft. The highest wave was 42 ft; heights greater than this are experienced infrequently. In other published results (see Putz 1952, Seiwell 1948, Watters 1953) wave-height ranges were much smaller.

It is interesting to list the values obtained for $H_{\frac{1}{3}}/H_m$:

Putz 1·63, Seiwell 1·57, Watters 1·58, Longuet-Higgins (theoretical) 1·60, this study 1·60.

It thus appears that Longuet-Higgins's predicted value is confirmed to a high degree of accuracy.

5. Maximum wave height

In practice the maximum wave height is of great importance. In Fig. 1 (b) H_{max} has been plotted against H_m. There is more scatter than in Fig. 1 (a) partly because, whereas H_m and $H_{\frac{1}{3}}$ were calculated to two decimal places, individual waves (and so H_{max}) were measured to the nearest foot; in addition H_{max} might be expected to be a less ' stable ' parameter than an average such as H_m or $H_{\frac{1}{3}}$.

Nevertheless the plot is almost linear. The best-fit straight line through the origin is

$$H_{max} = 2{\cdot}398\, H_m$$

and

$$\rho_{H_m H_{max}} = 0{\cdot}905$$

It follows that

$$H_{max} = 1{\cdot}50\, H_{\frac{1}{3}}$$

Longuet-Higgins showed theoretically that the ratio H_{max}/H_m should depend upon the number of waves in the record, and deduced a ratio 2·32 for 50 waves and 2·51 for 100 waves. The mean number of waves per record in the present series was 69, and the mean ratio 2·40. In spite of this apparent confirmation of Longuet-Higgins's result, in the present investigation there is no systematic relationship between n (the number of waves), and the value of H_{max}/H_m; perhaps this is to be expected as the number of waves varies within comparatively narrow limits.

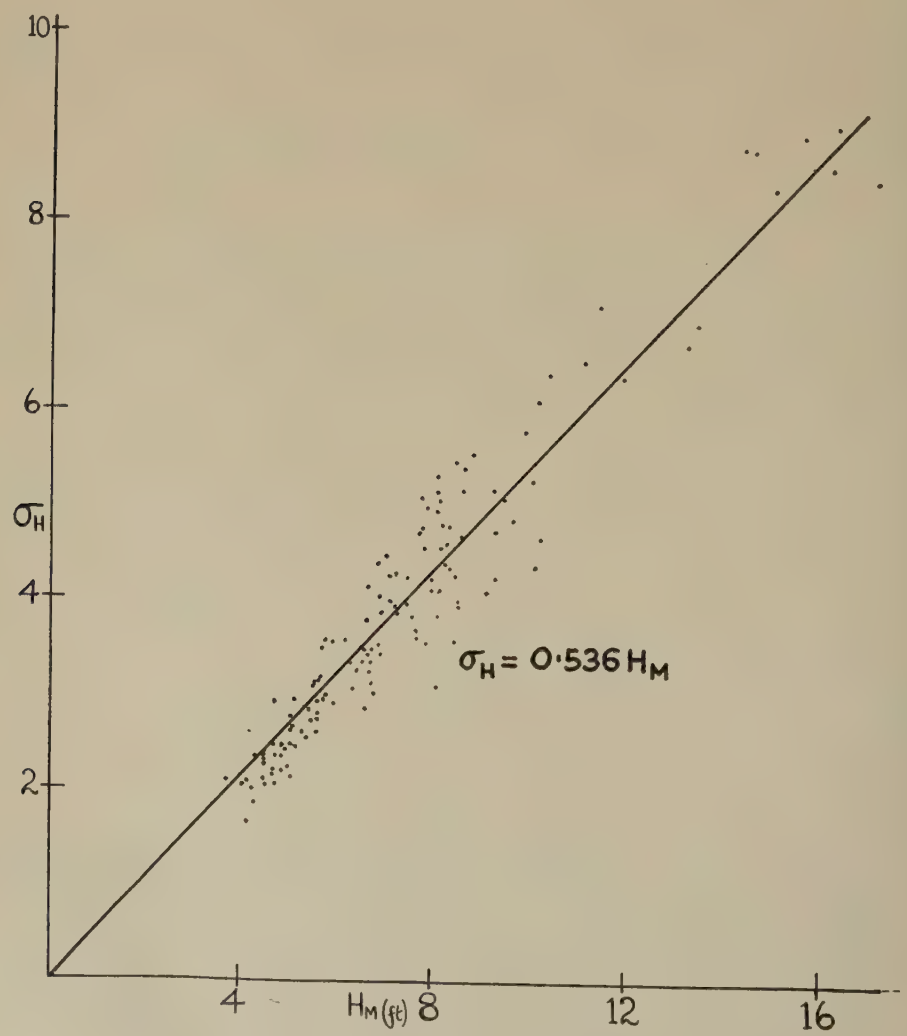

Figure 2. Standard deviation of wave height against mean wave height.

6. Dispersion of wave heights

From the foregoing discussion it is clear that a knowledge of any one of H_m, $H_{\frac{1}{3}}$ or $H_{\max}$ enables the others to be estimated with considerable accuracy. It is also important to be able to estimate the wave-height dispersion from any one of the above parameters.

In Fig. 2 σ_H is plotted against H_m. The best-fit straight line is given by

$$\sigma_H = 0{\cdot}54\, H_m - 0{\cdot}05$$

and $$\rho_{\sigma H\, Hm} = 0{\cdot}928$$

Physically, of course, σ_H and H_m tend to zero together, so the smallness of the constant is encouraging. The best-fit straight line through the origin is given by

$$\sigma_H = 0{\cdot}536\, H_m,$$ and this line has been drawn on Fig. 2.

Longuet-Higgins gave the theoretical value

$$\sigma_H = \left(\frac{4-\pi}{\pi}\right)^{\frac{1}{2}} H_m$$

i.e. $$\sigma_H = 0{\cdot}522\, H_m,$$ in good agreement with the above result.

Putz obtained $\sigma_H = 0{\cdot}494\, H_m + 0{\cdot}120$, but his range for H_m was small, $\frac{1}{2}$ ft to 5 ft. A good working rule is to take a standard deviation equal to half the mean wave height.

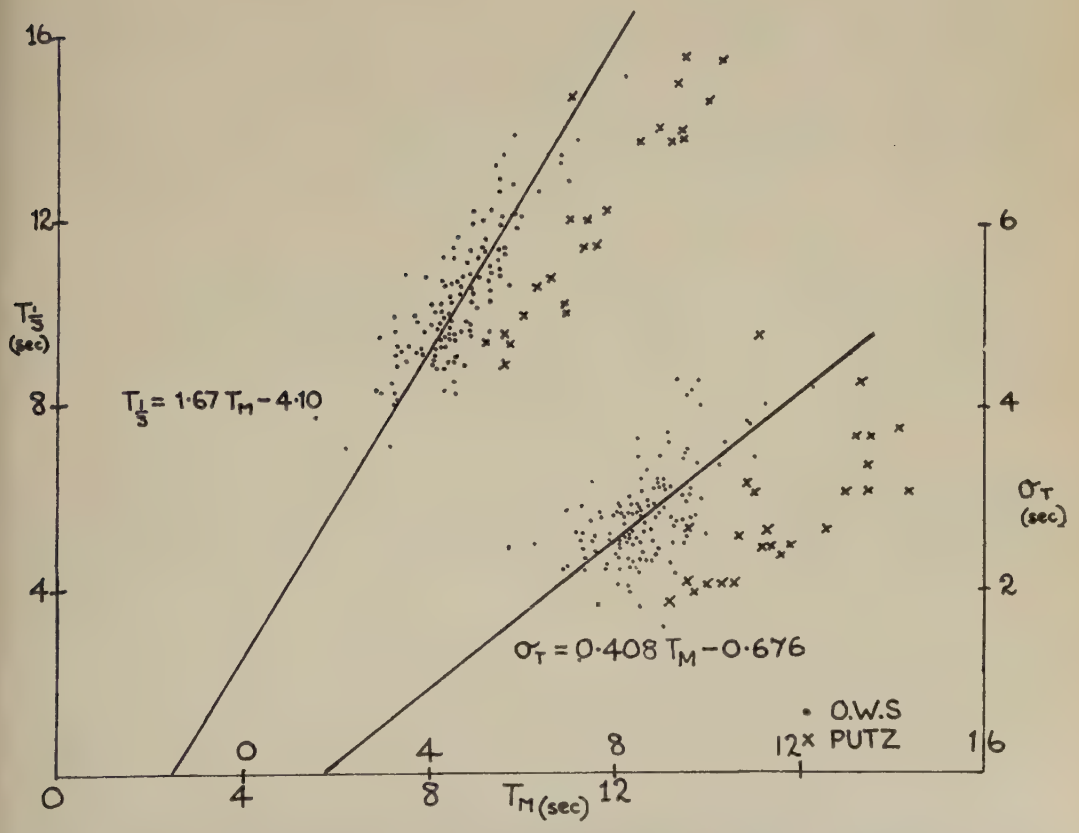

Figure 3. (a) Mean period of the highest third of the waves against mean period. (b) Standard deviation of wave periods against mean wave period (abscissa shifted to right).

7. Wave periods

The mean of the periods of the highest third of the waves was also found and Fig. 3 (a) shows $T_{\frac{1}{3}}$ plotted against T_m. The periods vary between 5·5 sec and 12·5 sec for T_m, and between 7 sec and 15 sec for $T_{\frac{1}{3}}$. For comparison the observations by Putz are also plotted.

The best-fit straight line is given by

$$T_{\frac{1}{3}} = 1{\cdot}67\ T_m - 4{\cdot}10$$

and $$\rho_{T_m T_{\frac{1}{3}}} = 0{\cdot}851$$

The relation obtained by Putz is rather different: he gives $T_{\frac{1}{3}} = 1{\cdot}25\ T_m - 2{\cdot}29$. His periods vary between 9·2 sec and 14·4 sec for T_m and between 8·9 sec and 15·7 sec for $T_{\frac{1}{3}}$. Generally, as T_m and $T_{\frac{1}{3}}$ must tend to zero together, the above results suggest that the relationship between them is non-linear.

It is worth noticing that both the above equations imply that for short periods $T_{\frac{1}{3}}$ should become less than T_m. The first equation gives $T_{\frac{1}{3}} < T_m$ for $T_m < 6$ sec, while Putz's equation would make $T_{\frac{1}{3}} < T_m$ for $T_m < 9$ sec. Neither set of results contains values of T_m below its own critical value, and neither equation can be used for extrapolation.

A possible reason for the near equality between T_m and $T_{\frac{1}{3}}$ over the range of periods observed by Putz is the damping out of short-period waves by bottom pressure-recording apparatus. This would tend to raise T_m without affecting $T_{\frac{1}{3}}$ very much.

8. Wave-period dispersion

Fig. 3 (b) shows σ_T plotted against T_m. The correlation is much lower than for other pairs of variables: $\rho_{\sigma_T T_m} = {\cdot}583$.

Putz's values are again plotted. The best-fit straight line is

$$\sigma_T = 0{\cdot}408\ T_m - 0{\cdot}676$$

Putz gives $$\sigma_T = 0{\cdot}313\ T_m - 0{\cdot}759$$

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A note on computed transports of angular momentum in a theoretical model of the general circulation

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SUMMARY

Angular momentum transports due to mean meridional circulations and horizontal exchange processes are computed for a theoretical model. It is found that, in high latitudes, the transport due to mean meridional circulations is less than 1 per cent of that due to horizontal exchange processes. In the tropics, however, the transport due to mean meridional circulations is seen to dominate that due to horizontal exchange processes. It is suggested that a variable coefficient of eddy viscosity would have to be used in order to explain observed motions on a synoptic scale.

In a recent publication the author (1954) has calculated the velocity fields for a model of the general circulation of the atmosphere. The essential features of the model are zonally uniform heating, incompressibility and the use of a constant coefficient of eddy viscosity. The velocities thus obtained were steady and zonally uniform. It was found that the effect of including the non-linear terms in the equations of motion was to produce a belt of easterlies at low latitudes, corresponding to the trade-wind belt, and a region of maximum subsidence at 20°N. The position of maximum westerly velocity, the sub-tropical jet for the model, was found to be at about 10°N.

Using these velocity fields, incorporating the non-linear terms, it is of interest to compute the rates at which angular momentum is transported firstly by meridional cells and secondly by horizontal exchange processes, allowed for by a coefficient of eddy viscosity, and to compare these quantities. The relative angular momentum transport due to mean meridional circulations has been averaged with respect to the height, by computing the integral

$$I_1 = a \sin\theta \int_0^1 \rho u v d\zeta$$

at different latitudes. In this expression

a = radius of the earth
θ = co-latitude
u = N $\rightarrow$ S velocity
v = W $\rightarrow$ E velocity
ρ = fluid density

and ζ is a non-dimensional vertical co-ordinate such that $\zeta = 0$ represents ground level, and $\zeta = 1$ the tropopause for the model. This integral simply represents the rate at which the relative angular momentum of a unit volume of fluid is transported southwards. The results are shown below :

Latitude	10°	15°	20°	25°	30°	60°
I/a dynes/cm^2 (+ towards south)	− 5·15	− 0·299	0·086	0·058	0·048	0·0006

This table brings out two significant facts. Firstly, the largest transport due to mean meridional circulations takes place in the tropics, where it is directed towards the north. Secondly, the transport in all higher latitudes is very much less but is southwards, so that in each case the direction of the momentum transport is towards the sub-tropical jet, with the exception of latitude 15°N.

The eddy transport of angular momentum can be evaluated in a similar manner from the integral

$$I_2 = \rho K a \sin\theta \int_0^1 \frac{\partial v}{a\partial\theta}\, d\zeta$$

which gives the moment of the stresses about the axis of rotation. In this expression, K is the coefficient of eddy viscosity, which was taken to be 10^5 c.g.s. units in working out (Rogers 1954) the velocity fields. Using this value again gives the following table for the eddy transport of angular momentum.

Latitude	10°	15°	20°	25°	30°	60°
I/a dynes/cm^2 (+ southwards)	+ 0·549	− 0·033	− 0·069	− 0·084	− 0·089	− 0·090

The positive sign for the value at 10°N indicates that the transport is southward; elsewhere it is northward, i.e., down the velocity gradient and away from the sub-tropical jet at all latitudes. A comparison of these two tables reveals that the momentum transport due to mean meridional circulation is roughly ten times as large as that due to eddy processes in the tropics, so that there is a net transfer northwards. At 20°N the two transports are roughly equal, while at 60°N the eddy transport is over one hundred times as large as that due to mean meridional circulations. The relative amounts are shown in the following table :

Latitude	10°	15°	20°	25°	30°	50°
(Circulation transport)/(eddy transport)	9·39	9·11	1·25	0·69	0·52	0·007

Starr and White (1952), however, in a study of the meridional flux of angular momentum in the tropics found that about 14 per cent of the total transport across 13°N is brought about by mean meridional circulations. The discrepancy can be explained by the arbitrary choice of the magnitude of the coefficient of eddy viscosity in the model. A possible way of determining this is to use a measured value of the stress. In his discussion of the general circulation of the earth's atmosphere Starr (1951) gives a value of 50-100 dynes/cm^2 for the northern border of the tropical easterlies where it is assumed, as in the model, that the main transfer of momentum takes place in the troposphere. Taking the lower limit it is found that this gives a value of 0 (10^7) for K. With this value for the eddy viscosity, the figures in the above table now represent the percentage ratio of the momentum transports. This, unfortunately, leads to the complication that the transport of momentum in the tropics is now southward since the transport due to mean meridional circulations is completely dominated by that due to horizontal exchange processes. A way out of this difficulty would be to drop the assumption that the coefficient of eddy viscosity is constant and allow it to vary in some fashion with latitude. The results indicate that it is doubtful whether the assumption of the existence of a constant coefficient of eddy viscosity is sufficient to explain observed atmospheric motions on a synoptic scale.

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551.551 : 551.558.21

Turbulence generated in standing waves

By R. S. SCORER
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(Manuscript received 12 July 1954)

SUMMARY

It is shown that if standing waves occur clear-air turbulence is to be expected at levels where there are discontinuities of shear in the airstream.

A recent encounter with exceptional clear-air turbulence on an occasion when standing waves were widely reported (Jones 1954) makes it desirable to give a fuller discussion of the possible connection between waves and the turbulence. If meteorologists and pilots are aware of this connection it will be easier for them to report significant observations.

Clear-air turbulence has often been reported by glider pilots soaring in standing waves, with smooth air above and below, and it was obviously not produced by cloud or by surface roughness. If a glider is trimmed to ascend in the upwind side of a wave so that it has almost no motion relative to the ground it is possible to detect small variations in the horizontal component of the wind, and Mr. A. L. Slater, of the Derby and Lancashire Gliding Club, has described to me how on ascending through a shallow layer of turbulence in this way the wind above the layer appeared to be 2 or 3 mi/hr greater above than below. A layer of turbulent air when waves were present was reported by Roper (1952). Many observations of turbulent clear air have been reported by sailplane pilots particularly of the Midland Gliding Club soaring in waves at the Long Mynd (for example, see *Gliding*, 1951-4 : R. C. Forbes, **2**, p. 22; R. H. Prestwick, **3**, p. 163; S. H. Georgeson, **3**, p. 178, **4**, p. 67; S. Jones, **4**, p. 130; A. Pickup, **5**, p. 16).

In describing reports of pilots of powered aircraft at great heights much emphasis has been placed on the presence of a jet stream and the large horizontal and vertical shears observed in the neighbourhood. The presence of a jet stream is usually favourable to the formation of mountain waves : turbulence has also been observed over the ocean many hundreds of miles from mountains but in the neighbourhood of a jet stream, and it seems likely that there are two ways in which the turbulence may be associated with it (as suggested by Scorer 1952). Here we are only concerned with the creation of turbulence through the agency of mountain waves.

In the presence of solid boundaries it is known that flow of a viscous fluid is unstable when the shear is large and that the boundary layer often becomes turbulent, but no estimate of the magnitude of the shear required to produce instability has proved satisfactory. In particular the existence or otherwise of turbulence has failed to show any close association with the local Richardson number of the flow, and in the absence of solid boundaries a mechanism must be found which will produce large shear in the airstream if we are to ascribe the presence of the turbulence to instability due to large shear.

Mountain waves have lengths, typically, in the range 1-8 mi. The air passes continually through an almost stationary wave pattern, so that if, in so doing, shear is created such shear is continuously being created in new air and if the shear is large enough to be unstable then turbulence can be expected to appear in, or slightly downstream of, the places where the shear appears. The turbulence presumably reduces the shear but it can endure because new air is continually fed into the region. In particular we may examine what kind of airstreams will contain *discontinuities* in the perturbation of horizontal velocity (u). This is the extreme case : it is interesting because discontinuities of horizontal velocity are known to be dynamically unstable even if the static stability might prevent regions of large shear from being unstable. We therefore suppose that the basic air current contains no discontinuities of its horizontal velocity (U). The vertical displacement (ζ) is a continuous function of height (z), and so is the pressure perturbation (p). According to Scorer (1949, Eq. 5), neglecting the earth's rotation

$$p = -\rho_0 \{Uu + U' \int w\, dx\} \quad . \quad . \quad . \quad . \quad (1)$$

where a prime denotes $\partial/\partial z$, ρ_0 is the undisturbed density, w the vertical velocity, and x the

coordinate in the direction of motion. But $U\zeta = \int w\,dx$ and so

$$p = -\rho_0 U\{u + U'\zeta\} \quad . \quad . \quad . \quad . \quad (2)$$

As we have just seen, if there is no discontinuity of temperature then all the quantities in (2) are continuous functions of z except possibly u and U'. If U' is discontinuous then so is u. Therefore in standing waves discontinuities of horizontal velocity will appear at levels where the shear of the basic current is discontinuous. In the less extreme case, at levels where U'' is large, $\partial u/\partial z$ is also large. This is interesting because two writers (Trefry and Parker, see 'O.S.T.I.V.' 1952; meteorology and gliding,' *Q.J.*, **78**, p. 635) had found an association between large values of U'' and the occurrence of clear-air turbulence; but they had been led to expect such an association by noting that in the equation for the variation with height of the vertical velocity in standing waves the significant coefficient was

$$g\beta/U^2 - U''/U$$

commonly denoted by l^2. Here β is the stability $\left(\frac{1}{\theta}\frac{\partial\theta}{\partial z}\right.$ where θ is the potential temperature$\left.\right)$ and it appears only in this coefficient. Their suggestion was that if U'' were so large that $U''/U > g\beta/U^2$ the effect of the static stability would be nullified and the air would behave as if it were statically unstable and so turbulence would result. There is nothing in the theory of standing waves to suggest that the air is unstable if l^2 is negative; and since the instability is entirely internal in the stream the velocity, U, of the air relative to the ground cannot be significant, yet the sign of l^2 can be altered by subtracting from or adding a constant to U.

From Eq. (2), however, we may calculate the magnitude of the discontinuity in u. We note first that it is in phase with ζ so that it is a maximum at the crests and troughs of the waves, except where the waves are not stacked vertically. Also

$$\Delta u = -\zeta\,\Delta U' \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where Δ denotes the amount of the discontinuity. (This is Eq. (40) of Scorer 1953, which was derived somewhat differently.) This is equivalent to saying that if there is no sharp discontinuity but a shallow region of large shear at a level where U'' is large then

$$u' = -\zeta U'' \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In what might be a typical case the vertical semi-amplitude of the wave might be ½ km, and the discontinuity of the shear about 10^{-2} sec^{-1}, then the discontinuity of u in the wave crest or trough would be 5 m sec^{-1}, which is very considerable.

In analysing the airstream in any reported case of clear-air turbulence it is therefore important to look for discontinuities in the value of U'. It may not be easy to find these in ordinary radiosonde data, and it is obviously better to examine the actual data transmitted by the instrument than the necessarily abbreviated coded message. In calculating U'' a triple differentiation of the horizontal displacement is necessary, but it is reasonable to suppose that if two layers of air have a different type of velocity profile they also have different temperature profiles because subsidence is likely to be different in the two layers since it is closely related to the shear. Therefore we can expect the lapse rates to differ in the two layers. Discontinuities of lapse rate are more easily recognized than discontinuities of U' because they involve one less differentiation of the original data, and so it is worth while to see if they are associated with clear-air turbulence when standing waves are present.

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551.5 : 797.5

O.S.T.I.V. 1954; Meteorology and gliding

The biennial congress of the Organisation Scientifique et Technique Internationale du Vol à Voile was held this year at Buxton, from 22 July to 2 August, while the World Gliding Championships were taking place at Great Hucklow, nearby.

This year the papers were classified as meteorological or technical, and the meteorological ones occupied the first three days of meetings. They are described here.

Convection

The session on Convection was opened by R. S. Scorer (1954a) who showed how the atmosphere can be thought of as being stratified, when convection is active, into various layers – the surface layer, the super-adiabatic layer, the adiabatic layer, the sub-cloud layer, and the cloud layer. The properties of these layers were deduced qualitatively from the bubble model. The importance of downcurrents either between clouds or between large areas containing cloud was emphasized. Then W. C. Swinbank described the ' open parcel ' model of C. H. B. Priestley (1954) and explained the concepts of ' oscillatory ' and ' asymptotic ' motion and ' absolute buoyancy.' The behaviour of a parcel depends upon its size, but it was not clear how the size could be thought of in terms of observable elements of convection.

H. von Tippelskirch showed by means of experiments in liquid sulphur, carried out by himself and H. Koschmieder, that the direction of flow in a convection cell in a layer of fluid depends, as Graham had suggested, upon whether the viscosity increases or decreases with increasing temperature. Liquid sulphur, unlike most liquids, has a viscosity increasing with temperature in the range 119°C to 200°C. Below 119°C the cells have upward flow in the middle, but as the temperature is increased the flow ultimately becomes gas-like, with down motion in the middle of the cells. The motion was made visible by aluminium powder, and the liquid-like flow shows predominantly lines of convergence on the upper surface while gas-like flow shows points of convergence.

The question whether soaring flights could be made across the ocean, particularly in the equatorward-moving air masses of the trade winds, was discussed by R. Maletzke. There being no important diurnal variations in the convection, flights of long duration were optimistically contemplated. Some doubts were expressed when he interpreted Woodcock's observations on soaring birds as proving that the updraughts could be exploited by a glider.

The conditions necessary for the formation of cloud-streets were discussed by J. P. Küttner. A wind maximum at about cloud-base level (or about the middle of the convection layer) and a definite upper limit to the convection, usually a subsidence inversion, are most favourable. These are produced typically on the east side of a cold high where the wind from the north decreases with height because of the thermal field, but increases with height in the friction layer. The wind structure is often similar in a hurricane in which, as radar pictures have shown, the precipitation is found in almost parallel lines spiralling in towards the eye. For theoretical justification he appealed to an extension of Rayleigh's classical treatment, in which, of course, viscosity is fundamental. The evident implications of his theory for the soaring pilot were explained. The same problem was treated from the synoptic standpoint by N. Gerbier who showed that the best conditions for long-distance flights in France were in north-easterlies with anticylonic isobars and convection in the layer below the subsidence inversion. For most favourable conditions this inversion should be between 1,500 m and 2,500 m above the ground. He also showed typical upper-air soundings accompanying this ' convective divergent ' flow. If heating from the ground failed, thermal flights could nevertheless be made if enough potential instability existed and there was convergence or orographic lifting to start it.

The attempt by J. Malkus and R. S. Scorer (1955) to obtain a quantitative description of the eroding-bubble model of convection was outlined. It was claimed that a formula they proposed had been verified by measurements of the rate of ascent of isolated cumulus towers from time-lapse pictures. The formula involved a constant, which represents the rate of erosion, and relates the size to the buoyancy, lifetime, and vertical velocity of a bubble. It enables the lifetime to be computed from the buoyancy and size.

A new theme was introduced into O.S.T.I.V. discussions by J. Sayer. After a brief introduction describing the life history of the locust, he presented a paper by R. C. Rainey. This described how swarms of desert locusts exploit thermals in their great migrations in which they

travel at a speed slightly less than the wind by an amount depending on the time of day, as they vary the proportion of the time spent in the air. Swarms assume stratiform or cumuliform shape according to the thermal activity. When they arrive over a large range of mountains the convergence from both sides often keeps the swarms in among the mountains for many weeks although they crossed the plains at perhaps 60-100 mi/day. Their behaviour gives a good indication of the air motion.

C. G. Johnson was also concerned with the transport of insects, aphids in this case, but, because it is impossible to see them in the free air, knowledge of their dispersal depends on the numbers caught in traps held on towers or balloon cables. Interpretation is complicated by the existence of two times of day when the number taking to the air is a maximum. Generally, however high they are carried by convection, they fall to the ground within an hour or two of sunset, but some so far inexplicable cases have occurred when aphids have been found in an isolated layer, at about 200 ft, in the morning. Clearly a greater knowledge of their reactions to the air and how the air treats them is required.

Thunderstorm electricity was discussed by B. J. Mason (1953a, 1953b) and J. P. Küttner. The former described the physical requirements that all theories must meet but that none had so far satisfied, and gave his own tentative theory; but Dr. Küttner was not convinced, though he agreed that graupel probably played a leading part in the charge-generation process. He described some new measurements made on the Mt. Washington Observatory which threw doubt on earlier ideas about charge signs in clouds. It was hoped that glider pilots might be able to obtain evidence about the sign of the charge on the precipitation and the nature of the precipitation in the charge generating region. Powered aircraft travel too fast to obtain unspoiled samples.

P. Temple (1954) gave a short talk about a ' thermal detector ' which employed thermopiles of very fine gauge wire. In principle he measures the temperature gradient in the direction of flight, using the motion of the glider, and across that direction by the temperature difference between wingtips. He claimed to be able, when flying as passenger, to direct the pilot into warmer, and therefore ascending, air.

Turbulence

The description of turbulence by means of the idea of the ' equivalent gust ' is unsatisfactory because the reaction time varies so much from one aircraft to another. P. B. MacCready therefore proposed to describe turbulence in terms of the fluctuations of horizontal velocity, because all aircraft react slowly to them and the reaction time is outside the range of periods of the fluctuations we are interested in. It is then necessary to obtain a characteristic curve which he called the ' impedance ' of the aircraft. This is different for different aircraft, and relates the fluctuations of horizontal velocity to the *g*-load experienced. If this relationship is once measured the *g*-loads imposed on an aircraft by any turbulence, specified in terms of its horizontal velocity fluctuations, can be computed. The proposal will only work if there really is a correlation between horizontal turbulence and *g*-load. This would certainly be true if the turbulence were isotropic (and the *g*-load results from the vertical turbulence) and Dr. MacCready argued that Kolmogoroff's similarity hypothesis leads one to expect atmospheric turbulence to be isotropic, in which case one would be concerned with eddies produced by degeneration, not the eddies of a size where energy of turbulence was being created. To test the usefulness of his idea it was proposed to fly a glider in several cases of turbulence with different eddy spectra to see whether the same impedance curve was obtained in all cases, as hoped. A glider was chosen on account of its high lift/drag ratio : this makes it very sensitive to vertical turbulence while the distance travelled in the time taken to react to a horizontal eddy is about the same as a heavy powered aircraft. He expected the ideas to apply to eddies of diameter less than 500 ft.

Waves

This session was opened by Dr. R. R. Long (1954). He used a Froude number to describe the flow for he was concerned with the flow over obstacles of finite size in stratified fluid streams of finite depth. He obtained an exact solution of the complete equations, for a particular case of density and velocity variation with height in which the equations are linear, and reproduced the predicted types of flow in model experiments. Although the model was not identical with the case in the theory it did not differ much. The obstacle moved along the base of the channel and the camera moved with it. The fluid was a salt solution of variable concentration and the flow was made visible by particles of aluminium. If the flow contains nodal surfaces then, as

the height of the obstacle is raised, the fluid velocity at these surfaces is increased to form jets with regions of very low velocity in between. When the height of the obstacle exceeds the height of the lowest nodal surface, blocking of the lower layers occurs and a disturbance of the flow spreads upstream, i.e., the oncoming stream is modified. For slower flow, ' rotors ' appear in the middle of the fluid, and for yet slower flow, waves occur. There was good agreement throughout between the theory and experiment. This work is to be regarded as fundamental fluid mechanics; though he was pressed hard to apply his results to the atmosphere Dr. Long pointed out that there was no exact analogy with the stratosphere at the top of his model and it was impossible to define a Froude number for an air current with great variations of velocity with height. Though there was some encouragement for perturbation theory in this work there were also some criticisms and the discussion tended to contrast the case where the mountains were small compared with the depth of the airstream with that in which they were large. Even so, perhaps on account of the difference in the upper boundary, no rotor motion analogous to that found in the middle of the layer has been observed in the atmosphere. In a short note P. Queney emphasized the importance of the shear and showed that, where it dominates the static-stability, a ' cat's-eye ' type of rotor motion could theoretically occur at levels where the fluid velocity is zero relative to the obstacle. When the velocity was zero at the ground the ' cat's-eye ' pattern was deformed to produce rotors of great magnitude on the ground with waves up above.

R. S. Scorer (1954b and 1954c) showed that waves have a maximum amplitude at or near to the level of maximum l^2 (for definition see references) and made special note of how inversions can be interpreted in the l^2-profile of an airstream. He also suggested a mechanism for generating clear-air turbulence in waves at levels of discontinuity of the shear in the airstream. G. A. Corby (1954) gave many illustrations of l^2-profiles, computed using C. E. Wallington's lee-wave scale (1953), for occasions where waves of considerable amplitude were reported. These gave support to the theory that l^2 must decrease with height. Cases when no waves were reported showed that the decrease was, on the average, about one-third of the decrease found in cases when waves occurred.

J. Zierep discussed the waves in a uniform stream of slowly increasing velocity. Waves appeared to be created at the upwind side of the disturbance and to move slowly through the pattern with a velocity much less than that of the wind.

Jet streams

The relation of jet streams to soaring was discussed in general terms by J. P. Küttner, who described the investigation of jet streams in America. The vertical and horizontal wind profiles seemed to be composed of two approximately exponential parts with a peak in between. Dr. Küttner saw the possibility of certain types of unstable motions above and below the jets. He interpreted the well-known cirrus bands and referred to the ' jetlets ' revealed by more accurate observations of recent years. He also referred to the Bishop wave whose best examples occur in jet streams. A. Gazzola discussed the synoptic situations which would produce good wave-soaring conditions in Italy. He was concerned mainly with an analysis of the positions of jet streams during 1952. Since the jet stream generally wandered considerably, and, under conditions most favourable for soaring, was from the north over the Alps, the usual meridional cross-sections were useless. His paper was read in English by a colleague, and referred to flights made by one of the Italian pilots competing in the Championships.

General

The latest version of the film of the Sierra Wave Project was shown : it contains many remarkable cloud photographs. Three time-lapse films of clouds were lent by the Munitalp Foundation. They showed growing cumulus, fast-moving cirrus and medium-level wave clouds. There was an outstanding example of ice streamers extending many miles downwind from lenticular wave clouds.

A new photographic theodolite has been developed which enables stereoscopic observations of clouds to be made, using a base line of one or two kilometres and radiotelephonic communication. H. Meyer claimed, in a paper by himself and H. Koschmieder, that cirrus can be plotted to within 5 m and the slopes of cloud filaments to within 1°. The computation is necessarily lengthy.

The discussions were made all the more valuable by the help of W. A. Eichenberger who spoke English, German, French, and Italian, and H. Merbt and J. P. Küttner who spoke English and German. They made it possible for a summary in German to be given *before* the presentation of papers in English, and in English *before* those in German.

Thanks are specially due to C. Slikkerveer of the O.S.T.I.V. Secretariat who, together with A. H. Yates, so successfully organized the meetings which took place in the Derbyshire Conference House.

The Board of the O.S.T.I.V. are grateful for the financial grant made by the Royal Meteorological Society towards the expenses of the meteorological part of the conference.

It is expected that those papers not already published or in course of publication will soon appear and a brief summary of them together with a note of where they are published will ultimately be distributed by the O.S.T.I.V.

R.S.S.

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Correspondence

551.551

A note on the two-dimensional equation of diffusion in the atmosphere

Reply to Discussion

By D. R. Davies

As Dr. Lettau indicates in his remarks on p. 434 of this volume the continuity equation (20) must be replaced by

$$Q = \int_0^\infty \left[\bar{u}\,\bar{x} + l^2 \left(\frac{\partial \bar{u}}{\partial z}\right)\left(\frac{\partial \bar{x}}{\partial z}\right)\right] dz,$$

when we introduce the shearing advection term into the expression for the flux $\overline{ux}$. However, the analysis can be carried through as in my paper, and we find no change in the value of the n_0 parameter or the cloud height and little change in the ground concentration. Substitution of equations (10), (12), (21) and (22) into the above corrected continuity condition yields the equation

$$Q = \frac{\bar{u}_1 A}{z_1{}^m (1+2m)} x^{n_0+(1+m)(1+2m)^{-1}} \left[\int_0^\infty f(u)\, u^{-m(1+2m)^{-1}} du + m(1+2m)k^2 \int_0^\infty f'(u)\, u^{(1+m)(1+2m)^{-1}} du \right].$$

The index of x is to be zero and hence $n_0 = -(1+m)(1+2m)^{-1}$ as in my paper; the second term in the brackets is due to Dr. Lettau's correction.

Using the analysis described in the paper it can be easily shown that the ratio of this correction term to the first term (the original term in the analysis) in the brackets is just $m(1+2m)k^2$, i.e., 0·038 in the conditions considered in the paper. The corrected figures for theoretical cloud height and ground concentration in the Calder solution with Lettau's term introduced and using the corrected continuity condition are then found to be 10·0 m and 3,720 mg/m^3, replacing 10·0 m and 3,610 mg/m^3 given in the paper, i.e., a slight (3 per cent) increase in the value of the theoretical ground concentration.

Sheffield University.
3 July 1954.

551.508.54

A simple mechanism to improve the sensitivity of a Dines anemograph

By R. C. Munro

In the July issue of the *Q.J.* (p. 447) Trollund discusses the friction of a pen rod on a Dines anemometer. While the adaptation described overcomes a fault in an existing instrument, stated to have been in operation since 1930, I submit that the method we adopt and have used for many years, ensures that side pressures are eliminated in manufacture.

The usual float assembly is balanced for vertical movement by the shot cup but we found, invariably, a tendency for the float to travel across a tank when the top plate and guide were removed. The clock drum was supported from the top ring so that a record could still be made.

By application of a single weight fixed after trial and error to the top surface of the upper cone of the Dines float, we now test every one to prove that with the central guide or collar removed, the float will give a vertical trace up to 40 mi/hr and return to zero also with a vertical trace. This shows that side friction is eliminated and with a controlled clearance between float rod and collar we have no trouble whatever with zero settings or loss of record at low readings.

Quite recently an anemometer (P.T.A.) was reported to be giving long horizontal lines on the velocity scale. Knowing that the site was on timbers high up in the roof of a municipal building we suggested the tank had become out of level. The observer found this was so and after correction the horizontal lines did not repeat.

For our own instruments we would prefer to balance a float in the manner described instead of using steel ball bearings in accordance with the Chilian design.

R. W. Munro Ltd.,
Cline Road, Bounds Green,
London, N.11.
17 August 1954.

551.574.1 : 551.578.11

Observations of rainfall from warm clouds

By W. A. MORDY and L. E. EBER

(Read 19 May 1954. See *Q.J.*, **80**, p. 48)

MR. B. J. MASON : This paper which presents a first-rate piece of observational meteorology is most welcome; information of this kind is badly needed and I hope that Mr. Mordy will be encouraged to provide us with more.

His observation that the rainfall from the trade-wind cumuli was often associated with the appearance of an overhanging orographic canopy is most interesting. While it may be possible for droplets of several tens of microns, or even larger, to grow in this cloud sheet and seed the underlying cumuli, there is a second possibility. Those large droplets which develop inside the cumulus and are carried to the top and left behind by the evaporation of a cloud turret, will not evaporate if they enter the cloud canopy or the damp air below it. They may therefore survive, to continue their growth in a new turret and finally emerge as raindrops. The absence of the orographic cloud probably denotes very dry air above the cloud tops and the probability that any large droplets left stranded in it will evaporate.

I think that Mr. Mordy is to be congratulated on having the good sense to work in a region where there is a wide variety of interesting phenomena which occur regularly day after day. This must be a great advantage, and I confess to being a little envious.

MR. F. H. LUDLAM : It appears from the films that the layer cloud above the overland cumulus may arise either independently of the cumulus or, partly at least, by the spreading out of cumulus tops. Would Mr. Mordy confirm that the large drops he described, falling from the leeward side of the layer cloud, occur when it is formed independently of the cumulus ?

MR. J. W. ARCHBOLD : When Mr. Hamilton and I were in Nigeria, pilots reported that, flying at 10,000 ft, they were ' above everything,' while at the same time the charts were indicating showers. This means that our cumulus clouds, giving rain, were about the same size as yours. It is surprising to find that your clouds seemed to need a canopy while ours did not.

DR. R. C. SUTCLIFFE : Would Mr. Mordy tell us if the weather he describes was accompanied by electrical phenomena ? It would also be interesting to know whether radar was used to study the onset of precipitation as this might show the connexion with the lenticular-cloud layer.

DR. D. G. JAMES : I wonder if the large droplets reported by Mr. Mordy in the tops of the cumuli and in the stratified cloud canopy could have been produced by the large rate of forced ascent of the cloud air. A rough estimate from the film shown indicates a forced rate of ascent of some 30 to 40 ft/sec at the cloud tops. Is it not possible that the degree of supersaturation obtained is sufficient to produce the abnormally large droplets ?

DR. R. S. SCORER : In pursuance of one of Mr. Mason's points, the smooth lenticular cloud above the cumulus over the mountains which occurs when precipitation is large, but is absent when there is no precipitation, may not actually supply part of the precipitation mechanism but may merely be evidence of high humidity at cumulus-top levels. As Mr. Ludlam has pointed out in one of his papers, if some of the larger droplets in the cumulus-tops can survive when the cloud evaporates they can fall back into succeeding clouds and start the growth of precipitation : this is encouraged by high humidity.

MR. L. C. W. BONACINA : I should like to ask whether such enormous quantities of orographic rainfall in Hawaii are associated with a high inversion layer, that is to say, with a great depth of moisture-laden trade wind.

PROF. P. A. SHEPPARD : In a recent expedition in spring 1953 to Anegada (18°45′N, 64°20′W) a low, flat island about 10 mi × 2 mi, in the British Virgin Islands, we observed a number of phenomena very similar to those reported by Mr. Mordy but without the complication of topography. In particular shower rainfall, with rather large drops though of no great intensity, commonly fell from trade cumuli of only a few thousand feet depth, with tops by rough estimation much warmer than 0°C.

I was particularly interested in Mr. Mordy's remarks about the diurnal variation in precipitation on and around an island in the trades. In Anegada a typical diurnal progression in trade cumuli was from a pepper-pot distribution of cumuli over and around the island in the night and early morning hours to a suppression of cumuli by mid-morning in a ring a few miles wide around the island and one dominating cumulus line running from a point over the middle

of the island in the downwind direction. This island-produced cumulus street commonly caused apparently heavy rain a few miles downwind of the island, but not on it. Isolated rain-bearing cumuli advancing on the island from upwind generally failed to surmount the island barrier of cumulus suppression. Was any similar mechanism present at Mr. Mordy's site? The topographical triggering of cumulus at Oahu, in addition to what we may call thermal triggering as experienced on Anegada, could presumably change the nature of the diurnal variation.

Mr. W. A. Mordy (*replying on behalf of both authors*): In reply, I should first like to say that I very much appreciate Mr. Mason's kind remarks. Concerning the canopy, or veil-like cloud, while undoubtedly effects such as described by Mr. Mason exist, the phenomenon that we were particularly interested in, occurs independently of the mingling of cumulus tops with the stratiform cloud. So I should say that these large drops can and do develop whether or not there are large drops contributed from the cumulus arising from beneath.

Mr. Ludlam's observations are correct. The canopy cloud does develop, either independently or in some instances by the spreading out of cumulus tops. However, the large drops which appear in the canopy cloud are most often observed when it is of very large horizontal extent and may or may not be associated with the cumulus clouds.

In reply to Dr. Archbold, I did not mean to give the impression that the canopy was a necessary condition for rainfall, but that it was a frequent condition during orographic rainfall. We often see in Hawaii the occurrence of trade-wind showers from isolated cumulus out over the sea. I am glad of an opportunity to correct this impression.

In reply to Dr. R. C. Sutcliffe, thunderstorms are very rare in Hawaii and occur only, in my observations, when the trade winds are interrupted and we have towering cumulus or cumulo-nimbus clouds, clouds which reach as high as 30,000 ft. We have no direct measurements of electrical phenomena in Hawaiian clouds. It has been suggested that electrical effects could play an important part even though electrical discharges were not often seen. I think I should simply have to answer this, 'I don't know.'

Concerning the second part of your question, we have at the institute two 10-cm radar units which we have used in connection with our studies. Unfortunately, these sets will not give us a return unless we have droplets at least 1 mm in diameter. Since we are dealing with droplets in the veil, or canopy, cloud which are of the order of hundreds of microns, we are not able to study these with our present radar equipment. One of the radar units is now being modified to operate on a 3 cm wavelength and we hope that we shall be able to make some studies of this kind with this unit.

Dr. James's estimate of the rate of ascent near the mountains would certainly be conservative. However, the rate of ascent in the stratified cloud must be a great deal less than this, because the tops of the stratified cloud, which normally are right at the base of the trade-wind temperature inversion, seldom vary in height more than 100 to 200 ft from the leading or forming edge to the crest of the cloud over the crest of the mountains. Consequently, I think that the best estimates that we could make of the rate of ascent at present would indicate that they are not sufficient even to support the size of drops which apparently we have. We hope to get better information about these factors shortly.

It is certainly true, as Dr. Scorer remarks, that the appearance of the stratiform cloud is associated with high humidity at the base of the inversion. It is also true that this cloud has large droplets in it which apparently almost move with the stream lines at this altitude.

As to the role that these droplets play in the cloud, or their origin, we can only speculate at the moment. Dr. Scorer may be quite right in his suggestion; however, we must wait until we have more information before this matter can be cleared up.

In reply to Mr. L. C. W. Bonacina, the depth of the moisture-laden air beneath the trade-wind inversion is certainly one of the strongest factors in the formation of rainfall in Hawaii. However, in spite of this we have a great variation in rainfall conditions from day to day when, for all intents and purposes, meteorological conditions seem similar. It is for this reason that we feel we must know much more about the details of rain formation as suggested in this paper before we can adequately explain our day-to-day rainfall variation.

The effect which Professor Sheppard describes is very clearly seen on some of the Hawaiian Islands, particularly Molokai and Lanai; in fact, these showers which rain a mile or two off-shore have a special name in Hawaii. They are called 'Naulu.' In the region in which the study was made, however, these effects are not recognizable. In fact, during the whole study, it was our observation that the clouds moved in from the sea and mingled with the orographic cloud, greatly complicating the pattern of rainfall on any particular day.

551.574.1 : 552.577.11 : 551.551

Turbulence in clouds as a factor in precipitation

By T. W. R. EAST and J. S. MARSHALL

(Read 19 May 1954. Presented by Prof. P. A. Sheppard. See *Q.J.*, **80**, p. 26)

Dr. I. C. Browne : From this paper, I gather that the components of turbulent velocity which have the greatest effect in promoting coalescence are those lying in a range of frequencies around 100 c/sec. To give an acceleration comparable to that of gravity, these velocity components must have amplitudes of the order of 1 cm/sec. I should like to ask the authors how realistic such a spectrum is – is it likely to be encountered in cumulus cloud ?

Mr. B. J. Mason : Considering the simple case of a sinusoidal applied motion, then in order to cause an appreciable increase in the frequency of collision between droplets of radii 15 μ and 10 μ, the air must oscillate with an amplitude of about 1 mm (the average spacing of cloud droplets) at a frequency of 100 c/sec. This would correspond to the propagation of an intense sound wave, the creation of which is hard to imagine in a cloud except, perhaps, during thunder. It is well known that smokes consisting of tiny droplets ($r \ll 1\ \mu$) contained in a closed tube can be induced to coagulate by the application of supersonic waves. In this case, little energy is required because the system is closed, standing waves are set up, and the air particles can undergo considerable displacement. In a cloud, where there are no walls, the energy required would be very large indeed. I recall that some attempts were made a few years ago in the United States to induce coalescence in a fog by using four very powerful sirens working at 400 c/sec. A slight improvement in visibility was reported over a distance of 300 × 75 ft but this technique was not considered a practical proposition.

Of course, it is more realistic to consider the air inside a cloud to be subject to random accelerations, but the turbulence spectrum centred about 100 c/sec must still contain sufficient energy to cause rms amplitudes of order 1 mm and rms accelerations of order g. Is it reasonable to assume that the large amounts of energy required can be dissipated in eddies of this frequency ? I am very sceptical, but it is a point which should be discussed by the authors.

Their case for invoking turbulence to speed up the early stages of growth by coalescence is based on Langmuir's values for the collision efficiency corrected for the finite size of the small droplet. But the reliability of these computations, for the case where drop and droplet are of comparable size, is highly uncertain. More accurate computations of the droplet trajectories indicate that the values of collision efficiency used by the authors may be serious underestimates. Consequently coalescence of droplets falling under gravity probably proceeds faster than they estimate and the necessity for turbulence is correspondingly reduced.

Finally, in warm climates, large (and presumably turbulent) cumulus clouds are observed to build up to considerable heights without precipitating. Their failure to do so is difficult to explain if turbulence is capable of initiating coalescence, but is perhaps understandable if large nuclei are required and these happen to be deficient.

It may well turn out that this paper will be instrumental in proving that turbulence does *not* play an important role in the coalescence of cloud droplets. If so, it will still have served a useful purpose.

Mr. G. J. Day : A recent paper by Sartor (*J. Met.*, **11**, p. 91) describes laboratory experiments which indicate that the catch efficiencies given by Langmuir and Blodgett are much too small for droplet diameters less than 20 μ to 30 μ.

These results would appear to obviate the necessity for turbulence in the coalescence process.

Dr. R. S. Scorer : It seems desirable to emphasize the extent to which the authors' conclusions depend upon their hypotheses. The supposition of random motion has led them to discuss it in terms of the Fourier components. In particular, when treated one-dimensionally, motion of this kind can only contain accelerations of magnitude $2g$ for very short periods and so the response time of the drops to accelerations of the surrounding air enters fundamentally into the computations. The motion is three-dimensional and the accelerations may easily be completely correlated in two directions. This would happen in any part of the motion that could be crudely described as a vortex. Instead of collisions having to occur because the large drops react more slowly to transient accelerations of the air, vortical motion would cause the acceleration to be applied radially for a relatively long time : the large particles would therefore

sweep through more air than under gravity alone because of the added radial velocity. This, of course, happens in the vortex thermometer, in which the larger drops are centrifuged outwards more rapidly than the small ones.

If one takes this picture of the disturbed motion in a cloud the statistical, or ' turbulence,' approach is inappropriate, because the motions in different directions are so highly correlated. Visual observations of the exterior of clouds reveals parts of the motion that can legitimately be thought of as individual eddies, and the vortical motions can often easily be seen to last for several seconds. For a simple vortex with velocity v at distance r from the centre a radial acceleration of $2g$ results if $v^2 = 2gr$. If $2r = 10$ m then $v = 10$ m sec^{-1}, and it is certainly not to be expected that in moderate-sized cumulus changes of velocity of 20 m sec^{-1} would occur in a distance of 10 m; but if $2r = 1$ m then velocity changes of 6 m sec^{-1} might occur – that is to say not that we have good reason to believe that they do occur but that if they did they could easily have passed unnoticed up to the present.

The important point is that without calling upon minute eddies, radial accelerations of $2g$ might well occur for long enough to produce coalescence. Vortical eddies with frequencies of 100 c/sec would be far from the size that is created in the cloud, and here it is worth while to examine the history of the motion in a cloud. It results from the static instability and large horizontal density gradients, and these are largest in those parts of the cloud where evaporation is most active. The centres of cumulus can be largely smooth to a glider while the edges are very bumpy, though a glider would not be sensitive to the eddies less than 1 m in characteristic size. Yet, if the stirring motion does originate at the same time as, and accompanies the evaporation it will not endure long enough to become either homogeneous or isotropic enough to be treated by existing theories; least of all can it be supposed that the decay process as normally envisaged is dominant. If turbulence existed with characteristic motions as large as are found in cumulus in the absence of spatial spreading it could endure for perhaps hours, while in the atmosphere the clouds evaporate and a stable stratification of the air results and the ' turbulence ' is destroyed in a few minutes. Thus it may be doubted whether the ' turbulence ' approach can be helpful.

I do not see how air motions of the kind discussed by the authors could be reasonably easily measured because there is no platform for the apparatus, but aircraft with directional hot-wire anemometers are not far from being able to discover whether eddies exist from about $\frac{1}{2}$ m upwards with the tangential velocities necessary to aid coalescence. If this aspect were investigated it would not be worth while to analyse the observations statistically for a turbulence spectrum, but a direct inspection of the spatial velocity variations measured should reveal whether, on the most favourable hypothesis, namely that the eddies are simple vortices, they are capable of being effective.

Yet even if there arises a prospect that the verdict will be against ' turbulence ' being an effective agent in inducing precipitation, it will be well worth while to establish the verdict beyond all reasonable doubt.

Mr. E. Gold : Do the authors give any grounds for supposing that the assumption $\sigma_A = g$ is not unreasonable ? Or do they make the assumption simply because they want to ?

Prof. P. A. Sheppard (*who presented the paper on behalf of the authors*) : While I shall leave the authors to make their own written reply to the discussion, I shall take this opportunity to remove some misunderstandings and point out what I believe to be misdirected criticism about the paper. Thus Dr. Browne and Mr. Mason appear to have confused the rms acceleration in the region of the spectrum around 100 c/sec with the total rms acceleration throughout the turbulence spectrum. It is the latter which, given a value of g, and for certain spectral density functions, produces the increased collision rates computed by the authors. Had Mr. Mason continued his simple harmonic calculation he would have found that an amplitude of 1 mm at 100 c/sec gives a rms acceleration between 1 and 2 orders up on g.

While Dr. Scorer has made an effective point he is a little less than fair to the authors, who have already made it in somewhat different terms. Thus, they are at pains to point out (*a*) that a three-dimensional spectrum should be considered, (*b*) that there may be substantial correlation between the component motions and (*c*) that a wave-number (spatial) spectrum would appropriately replace the spectrum of the motion at a fixed point in time. Whether one talks in terms of a simple vortex or of correlation in a turbulence spectrum is perhaps a matter of taste (the simple vortex has not many meteorological successes to its credit, particularly in the smaller scales of atmospheric motion) whereas the physical requirement in the present case is knowledge of the properties of the motions in a cumulus and these except in the isotropic part of the spectrum are likely to come first from observation rather than theory.

In making these remarks I do not wish to imply that I think there is reason at present to suppose that turbulence is an important agent in the initiation of the precipitation process.

Mr. T. W. R. East and Dr. J. S. Marshall (*in reply, communicated*) : Mr. Mason reminds us that the average spacing between drops is about 1 mm so that a drop must move this far to meet a droplet. But the drop is falling continuously under gravity and will meet with many droplets. The point is that such encounters will usually be too gentle to bring about coalescence unless turbulence can sometimes increase the relative velocity at the moment of collision. It is not necessary for the random part of the motion to be so great as 1 mm. Neither is it necessary that the rms acceleration of order g should be centred about 100 c/sec. If Fig. 9 of the paper were replotted as relative velocity of drop and droplet divided by air *acceleration* ($|U/A|$ instead of $|U/V|$) versus frequency, the rising portion up to 10 c/sec would become level, and the response curve would be falling steeply above 100 c/sec. This is why an acceleration of order g can be located at any low frequency or spread out over the whole low-frequency region below 100 c/sec and remain effective.

Viscous dissipation of turbulent motion rises steeply with frequency and makes the spectrum envisaged by Dr. Browne very improbable.

Returning to Mr. Mason's remark that such a spectrum corresponds to an intense sound wave, both radiated power and aural sensitivity fall off very rapidly with decreasing frequency, so a random spectrum nearer 1 c/sec would not be heard.

Mr. Mason suggests that ' coalescence of droplets falling under gravity probably proceeds faster than (the authors) estimate ': then how is it that ' large cumulus clouds are observed to build up to considerable heights without precipitating ' ? It is the observed stability of cumulus that supports Langmuir's zero or low efficiencies even for cloud droplets, and encouraged the authors to trust his results in a general kind of way, if not in detail.

Mr. Day calls attention to Sartor's experiments; a wholesale rise in predicted collection efficiencies is open to the objections of the preceding paragraph. Bearing in mind that the validity of the similarity procedure is not yet proven, however, the results are certainly of great interest to the present problem.

We agree with Dr. Scorer that, in the absence of gravity, vortical motion would be more effective than linear motion in producing collisions. However, for random acceleration comparable to gravity, the horizontal components are far less effective than the vertical. In terms of vortices, those with horizontal axes have greatest effect because at the lowest point of the orbit they are working with gravity, and such a motion is approximated reasonably well by resolving along the vertical axis. For rms accelerations much larger than gravity this no longer applies.

Dr. Scorer's estimates of possible orders of magnitude are useful. It must be admitted that we have introduced into our hypotheses quantities that are hardly amenable to measurement. An alternative to the use of instrumented aircraft would be to adapt the ' smoke-puff ' methods used in clear air by dropping a packet of metal foil into the cloud and measuring the diffusion and the average velocities of the strips of metal by radar. This method has been used by Warner and Bowen (*Tellus*, **5**, p. 36, 1953) for studying macroscopic motion in the atmosphere.

In reply to Mr. Gold, $\sigma_A = g$ was chosen simply to suit the calculations : it appeared by preliminary inspection of the problem that this was the least amount that would produce a substantial effect on rates of collision.

551.578.4

The terminal velocity of snowflakes

By M. P. LANGLEBEN

(Read 19 May 1954. See *Q.J.*, **80**, p. 174)

Mr. F. H. Ludlam : I think Dr. Langleben should have considered what effect a weak downdraught within a precipitation streak would have upon his conclusion that aggregation of crystals occurs at the generating level.

Mr. E. Gold : I should like to congratulate Dr. Langleben in putting before us most interesting information. I saw from the 16 frames he showed that the snowflakes appeared to move very uniformly and not at all like the dancing flakes we watch through a window. I was

led to this observation by my thought that the photographs of snowflakes ought to give some indication of turbulence in the atmosphere or at least of turbulence of the magnitude which would affect these comparatively large objects. Has Dr. Langleben any information on this aspect ?

Mr. L. C. W. Bonacina : I am a little perplexed as to the significance of the term ' terminal velocity ' of snowflakes when there is wind.

As Mr. Gold remarked, snow in a strong wind reaches the ground in a complicated dance, bobbing up and down, and to and fro, in a most bewildering fashion. Even when it does reach the ground it may be whisked away again, perhaps not to settle down finally, in exposed country, for miles.

Mr. B. J. Mason : Dr. Langleben infers that ice crystals coagulate to form flakes at temperatures well below 0°C on the grounds that their fall velocities aloft, as deduced from the slope of the precipitation streak and the wind shear, were of the same magnitude as the terminal velocities of snowflakes at the ground, but about 2 ft/sec^{-1} greater than that of ice crystals. I should like to ask him how accurately he considers he can deduce the fall velocities of the particles from the slope of the streak and from the wind data. I feel that aggregation of crystals at low temperatures, if it occurs, must do so under rather unusual circumstances. If it were a common occurrence, would not the A-scope of a vertically pointing radar show a marked increase of echo intensity with decreasing height above the melting level ? I have not observed this to be the case, but perhaps Dr. Langleben would care to comment on this.

Dr. I. C. Browne : The remarks Mr. Mason has just made are in agreement with the observations I have made of upper bands. Their apparent speed of fall has been consistent with the measured wind speed and shear, if the particles producing the upper band have a fall velocity of about 2 ft/sec. Such a terminal velocity is, of course, appropriate to ice crystals rather than to aggregates of crystals. The intensity of the echo from an upper band increases only slowly with decreasing height, in a way which suggests that the particles are growing only by a sublimation process.

Dr. R. S. Scorer : The only occasion I have entered a cloud producing snowflakes was during a visit recently to the Jungfraujoch observatory. At lower levels there were large flakes but at the Sphinx observatory in cloud at − 20°C the snow was in much smaller elements and the small drifts around the building consisted of a much finer ' powder ' than down below nearer the freezing level.

Mr. G. J. Day (*communicated*) : In my experience aggregations of snowflakes are not observed on aircraft windscreens at any great distance above the melting level.

Dr. M. P. Langleben (*in reply*) : Mr. Ludlam has raised an interesting point. I myself have suggested the existence of both updraughts and downdraughts in the generating elements of the precipitation streaks which may be considered as convective cells. It is improbable however that in cyclonic, or precipitation-producing situations, a weak downdraught could exist, particularly through a depth of say 10,000 ft below the generating level and over a horizontal range of 30 or more miles, the orders of magnitude associated with a snow precipitation streak.

In reply to Mr. Gold and Mr. Bonacina, I should like to state that a three-dimensional analysis of the trajectories of snowflakes was originally planned but was given up as being too tedious and not really necessary. Except under gusty conditions, snowflakes fall more regularly than one may perhaps think. I was content to make measurements under rather special conditions as explained in the paper. Surely the velocities measured are applicable to free-space fall, hence the term ' terminal-velocity.'

To answer Mr. Mason's questions, I think that errors in measurement of the slopes of the precipitation streaks from the film records were negligible because of the suitable height versus time (hence range) scales chosen. Canadian meteorologists feel that the errors in their upper winds are less than 20 per cent. The combined errors could not account for the 2 ft sec^{-1} by which the calculated vertical velocities exceeded those of ice crystals. As for his second query, I believe that the main aggregation of the ice crystals is taking place in the generating elements which appear to be turbulent. There is then little reason to expect a marked increase of echo intensity with decreasing height as Mr. Mason has suggested.

Perhaps Dr. Browne has been fortunate enough to have detected cirrus streaks on his radar, hence his low fall velocities. I would like to draw attention to the observations of Mr. R. F. Jones of the Meteorological Office (*Met. Mag.*, **79**, p. 198). In the same manner as Dr. Browne, he deduced a mean rate of fall of the snow particles between 17,000 and 10,000 ft as about 5 ft sec^{-1}. This seems to imply the presence of aggregates rather than of individual crystals.

As regards the comments of Dr. Scorer and Mr. Day, I would like to say this. In taking surface observations of crystal types I have noticed aggregates of snowflakes at temperatures considerably below 0°C. To digress a little, Canadian snowfalls can, I think, be divided into two groups which may be likened to convective showers and continuous frontal rain. The less frequent type is that which manifests itself on a range-height tube as a uniform echo extending up to about 10,000 ft. This can be attributed to a steady growth of ice crystals in saturated air throughout their fall. Little aggregation is to be expected here. The far more common case is when isolated convective elements form and it seems that large flakes are produced which result in the precipitation streaks. The occurrence of these generating cells is not as frequent over the British Isles. At any rate, I saw very few of them during the winter 1953-54 when I was at the Meteorological Office radar station at East Hill. A possible reason is that the air masses in England, because of the maritime influence, are not as clearly defined as in the Canadian interior.

551.509.317 : 681.14

(*a*) Computation of tendencies and vertical motion with a two-parameter model of the atmosphere

(*b*) The computation of forecast charts by application of the Sawyer-Bushby two-parameter model

By F. H. BUSHBY and MAVIS K. HINDS

(Read 16 June 1954. See *Q.J.*, **80**, pp. 16 and 165)

Dr. O. G. Sutton (*President*) : I have not long returned from Los Angeles, where I attended a stimulating conference on high-speed computing applied to weather prediction. Dr. Charney there discussed the famous storm of 20 November 1950, which was not predicted by the routine forecasters. He showed that a 2-level model also failed in this way, but with a 3-level model (900, 700 and 400 mb levels) there was marked cyclogenesis, although the centre was placed a little too far south. I cannot but believe that if the routine forecasters had been shown this chart in good time they would at least have paused before sending out a good-weather forecast. I would like to ask Mr. Bushby if he considers that an extension of his model to 3 levels would be likely to improve matters considerably.

As regards numerical forecasting as a routine operation, my own belief is that it holds out considerable promise. I think we should regard the mathematical prebaratic as a 'second opinion,' to be accepted or rejected by the duty forecaster as his knowledge of weather situations dictates. Let us hope that faith in the method will grow as results accumulate.

Prof. P. A. Sheppard : Can Mr. Bushby say what particular defect of the model is responsible for the exaggerated anticyclonic development in the predicted pattern ? Can he also say whether the effect of the vertical motion on the adiabatic change of temperature and so on the thickness tendency expresses itself in the forecast in any generally systematic way ?

Further, has Mr. Bushby dealt with any situation in which a cold, cut-off low has developed, and if so with what success ? I ask this question because the upper boundary condition is presumably least adequate on such occasions.

Dr. E. T. Eady : The model used by Sawyer and Bushby is very similar to other two-parameter models devised independently by quite a number of workers – perhaps because such models are a natural extension of earlier work by Sutcliffe. From a computational point of view all such models are identical, being governed by formally identical equations. Sawyer and Bushby find, consistent with the findings of Charney and Phillips and of Eliassen and Smeby, that prebaratic forecasts produced by numerical integration with this model are almost, if not quite as good as forecasts produced by subjective methods. While not, for a first attempt, discouraging, this makes one wonder whether the approximate equivalence of mechanical and subjective forecasts may not to a large extent be due to the fact that both get those features right which are easy to get right, simply by continuity. The superiority of the dynamical approach (which numerical methods attempt to follow) over the geometrical approach (which is the major

ingredient in ' subjective ' methods) should show itself when attempts are made to increase the detailed accuracy or the forecast period. However this presupposes not only that the model is sufficiently realistic but also that the process of computation reproduces with sufficient faithfulness the dynamical constraints. It is not sufficient that the computation scheme shall be stable in the sense usually meant when computational stability is mentioned. There are many ways in which errors, *due purely to the computational technique*, may build up and ultimately destroy the information. In principle it is easy to devise schemes which reduce such errors sufficiently but it is *not* easy to do so if the scheme is to be used in forecasting and computation time is limited. We should not assume automatically that present methods require only minor modifications, particularly when models more realistic than the two-parameter one are involved, in order to yield forecasting tools of maximum reliability and efficiency.

Dr. R. S. Scorer : I was glad to see that Mr. Bushby omitted from his presentation any calculation of the goodness of the forecast by his methods. This can be misleading and is in any case an improper test at this stage. The methods are being tested and learned and the most important thing is that a body of experience is being accumulated in this country.

Mr. B. J. Mason : I should like to ask Mr. Bushby about the effect of the boundary conditions on the forecast. Is it possible for rapid development on the boundaries of the area to upset the forecast seriously for the centre of the region within 12 or 24 hr ? Has Mr. Bushby tried the effect of putting the *actual* boundary conditions into the equations after the event and, if so, with what result ?

Mr. J. S. Sawyer : I should like to revert to Dr. Sutton's remark that calculations made by Dr. Charney and his collaborators in America upon a 2-parameter model of the atmosphere had indicated less cyclogenesis and anticyclogenesis than the calculations of Mr. Bushby and Miss Hinds. I think that this difference may be due partly to the different levels chosen by the investigators to represent the atmosphere. Dr. Charney took his initial data from the 700 mb and 300 mb levels whereas Mr. Bushby used 1,000 mb and 500 mb. The inclusion of data for the lower troposphere may be important.

It is perhaps worth remarking that the numerical analysis involved in Mr. Bushby's computations differs in some respects from the usual problems of numerical computation because the accuracy required in the solution is not high – as meteorologists we are satisfied with a result within a few per cent. The problem of obtaining approximate answers to these long numerical calculations with the minimum of work is important to numerical forecasting and I hope it will attract the attention of mathematicians.

Dr. R. C. Sutcliffe : I think it is useful to point out that the practical forecaster should not get the idea that these numerical methods introduce novel and difficult physical principles. For many years forecasters have tried to introduce dynamical concepts into their forecasts of the movements and developments of depressions and anticyclones; I suggested some years ago various rules relating developments with the vorticity patterns and most forecasters in this country have some acquaintance with these principles which have, I hope, been useful, they have certainly been interesting. The numerical methods are essentially methods of handling the same sort of ideas quantitatively and systematically and this must of course be regarded by all forecasters as an important step forward.

At this stage I should like to borrow an expression of the President's and suggest that if these methods are adopted for routine practice they should be regarded as giving ' a second opinion.' But it will not be the opinion of a colleague who may say for example that ' in my experience a depression of this type generally moves up the English Channel ' but the opinion of an objective colleague who always uses the same dynamical principles which have been explained to us beforehand. In course of time, especially if the methods can be further improved, the forecaster may learn that it pays to give the opinion a lot of weight and eventually may trust it completely.

For my own part I am not optimistic that the new methods will be able to cut out the glaring errors and uncertainties of short-period forecasting. I suspect that many new developments which vitiate extrapolatory methods will not be predictable by calculation either, because they are not determined by the present conditions as these are known to us. They are essentially of the nature of instability developments and even if we can read the signs there will be great uncertainty in precise location, timing and phasing of the new growth. But quantitative methods may of course in time lead to the quantitative treatment of probabilities where a definite prediction is unwarranted.

I share with Dr. Eady some optimism on the application of such methods to long-period forecasting because at present we have very little skill in this problem and any success will be

an advance. But the methods will almost certainly need to be changed. Mr. Bushby's equations as they stand, omitting as they do friction and turbulence and also omitting non-adiabatic processes, can be expected to have little value for a few days ahead.

Dr. J. S. Farquharson : Electronic computations based on the Sawyer-Bushby model certainly yield encouraging results and practising forecasters regard work of this kind as offering the best hopes of substantial progress in forecasting. Nevertheless Dr. Eady's suggestion that we should keep an open mind and explore other approaches to a solution of forecasting problems is a good one.

Working with upper-air data, e.g., in vorticity and divergence computations, it is possible to be deluded into thinking that a result obtained is significant for forecasting and it may be that the approximate agreement between actual and computed charts is due to the prevailing influence of the geostrophic effect. Substantial difficulties may yet have to be met in trying to improve on the results so far obtained.

Mr. E. Gold : I should like to ask Mr. Bushby how much of the difference between the actual results and the computed results is due to the inevitable error in the observations and how much to the approximate character of the equations. Is the process a ' chequer ' process, stepped out from a boundary ?

Mr. F. H. Bushby (*replying on behalf of both authors*) : In reply to Dr. Sutton's question, as to whether the use of a three-parameter model instead of the Sawyer-Bushby two-parameter model would improve our computed forecast charts, I do not think that we would find as much improvement as Dr. Charney did when he extended his model to three levels. One of the main reasons for the improvements in Dr. Charney's predictions was that the three-level model enabled more account to be taken of the lower layers of the atmosphere than the two-level model (700 and 300 mb levels). Our two-parameter model using 500 mb contours and 1,000-500 mb thicknesses already pays more attention to the lower atmospheric levels than the two-level model of Charney. Also, in our sector of the northern hemisphere, there are normally not the strong temperature gradients in the lowest layers of the atmosphere that one experiences over the American seaboard due to differential heating over land and sea.

The exaggerated anticyclonic development in some of the predicted charts mentioned by Professor Sheppard is probably due to the fact that two of the three basic equations refer primarily to stream functions, and only to contour heights after the geostrophic approximation has been used. In anticyclonic regions the two assumptions of geostrophic motion and the neglect of friction both cause errors of the same sign, but this is not so in cyclonic situations.

One situation has been examined in which a cold cut-off low developed. The cut-off process was forecast, but there was an error of about 200 mi in the position of the centre of the computed depression.

In reply to Mr. Mason, the size of the forecast region was originally chosen so that there would be little likelihood of the centre of the region being affected by rapid development on the boundary during the forecast period. However, it now seems likely that there are isolated occasions when boundary effects may have caused errors in the centre of the region, and it is the intention to carry out some computations using the correct boundary conditions.

In reply to Mr. Gold, I think that most of the difference between the actual results and the computed results is due to the approximate nature of the equations and not to errors in the original data. The problem of solving the Poisson and Helmholtz differential equations is a ' jury ' problem (i.e., one in which the answer is specified on all boundaries).

551.521.32 : 551.521.61

The evaluation of infra-red radiative fluxes due to atmospheric water vapour

By W. L. GODSON

(Read 16 June 1954. Presented by Dr. G. D. Robinson. See *Q.J.*, **79**, p. 367)

Dr. R. M. Goody : I would like to congratulate Dr. Godson on this interesting paper, and particularly upon his approach to this type of problem. It happens too frequently that approximations are made in radiation problems on the grounds of mathematical expediency,

whereas Dr. Godson has tested the accuracy of his approximations by trying them out on model atmospheres which are simple enough for the exact result to be computed.

There is one point which should perhaps be made. In the course of a discussion on a paper in the *Quarterly Journal*, Mr. A. R. Curtis (*Q.J.*, **78**, p. 638, 1952) pointed out that, as regards pressure effects upon the mean transmission, a good approximation is obtained by assuming that for an atmosphere with variable pressure,

$$\tau_I = \tau_I(m, \bar{p}),$$

where

$$m = \int dm,$$

$$\bar{p} = \frac{\int p dm}{\int dm}.$$

This approximation corresponds exactly to Dr. Godson's Eq. (8). From Tables 2 and 3 we see that it gives mean transmissions which are sometimes more accurate than $\bar{\tau}$ although on the average they are slightly worse. Curtis's approximation has the merit that its physical meaning is fairly obvious, and therefore under some circumstances it may be more useful than the one used in this paper.

Prof. P. A. Sheppard : Is there any physical basis, as opposed to mathematical convenience, for supposing that the Elsasser model of equally spaced lines of equal intensity will lead to useful results on flux and the divergence of flux ?

Dr. A. W. Brewer and Mr. J. T. Houghton : During the past year we have successfully made a number of measurements of upward and downward infra-red fluxes at various levels in the atmosphere up to 40,000 ft using a radiometer which we have described in the *Journal of Scientific Instruments* (**31**, p. 184, 1954). These measurements show that the charts of Elsasser and Yamamoto are more accurate at high levels than might have been expected. The difference between observed and calculated fluxes is about 10 per cent at the ground and up to 30 per cent at the highest levels. In general the differences are in the *opposite* sense from those reported by Dr. G. D. Robinson.

For ascents made on clear nights the divergences predicted by the radiation charts agree well with the observations. At the moment we are unable to measure divergence above the tropopause where the water vapour pressure is very low because the radiometer window absorbs strongly beyond 40 μ – the wavelength region where most of the divergence then occurs.

Clouds present a serious problem both for those who measure and for those who calculate the radiative fluxes in the atmosphere, for they greatly affect the field of the divergence and can occur in patches of any size at any level, so adding a complication which seriously limits the value of calculations or measurements.

Dr. R. S. Scorer : In discussing the apparent contradiction involved in requiring extremely high accuracy in computations with an admittedly crudely approximate model it may be noted that the accuracy is required on account of the mathematical operations to which the model is subjected, not on account of any necessity to simulate nature accurately. The calculation of divergences always requires an accurate knowledge of the basic field or fluxes, but two models that differ somewhat will probably lead to very similar results if they simulate roughly the same physical process.

Dr. G. D. Robinson (*in reply, partly communicated*) : I think the answer to Professor Sheppard's query is that Elsasser's model was set up because of its mathematical convenience, but that it was physically the most plausible of the mathematically convenient models available to its author. It is, for example, physically much sounder than the single-line model which is still used, and more flexible and convenient than Schnaidt's ' wingless ' single-line model, which was probably the soundest available approximation when Elsasser began his work. It seems to follow from Dr. Scorer's remarks, that so long as a model spectrum is not physically outrageous, mathematical convenience is the criterion of utility. This view seems to be widely held – I can think of no other explanation for the continued popularity of the single-line model – and may well be true. Dr. Godson's paper is however concerned with the next stage of computation – solution of the equation of transfer – and he has demonstrated the grave deficiencies of certain convenient methods.

We must welcome the measurements of Dr. Brewer and Mr. Houghton, and hope that they will prosper and be multiplied. I find their preliminary results rather puzzling, especially the agreement with the Elsasser chart in (as I understand) its original form with square root pressure ' correction ' and incorrect line width.

Dr. W. L. Godson (*in reply, communicated*) : I should like to thank Dr. G. D. Robinson for having presented my paper and for having so ably replied to the points raised in the discussion.

The purpose of this paper may be summarized, in retrospect, as follows : to investigate general methods of computation of transmissions for arbitrary atmospheres for the entire spectrum as well as for narrow spectral intervals (and in particular to assess the accuracy of simplified or empirical procedures), and to investigate the accuracy of flux computations, on radiation charts, when the temperature effects are handled in various conventional ways. From this point of view, it is natural to investigate the Elsasser model first, since it is realistic for certain atmospheric absorbents, although not for water vapour. Although only the regular model is applied specifically to water vapour in the present paper, similar results for flux errors on radiation charts, as a result of temperature effects, would be expected for a random spectral model.

Dr. Goody's remarks apply only to isothermal atmospheres; in the general case, the problem is considerably more complex.

Flux measurements at high levels, such as those of Dr. Brewer and Mr. Houghton, are very valuable. In view of the large number of sources of error when using existing radiation charts, I do not believe that either the sign or the magnitude of the errors could be predicted. The present paper discusses only errors due to temperature effects, when all charts use the same transmission functions and the same physical parameters.

Corrigendum to July 1954 issue

I. C. Browne *et al.*, p. 322, References—

Absalom, H. W. L. 1954 *Air Ministry, Met. Res. Cttee.*, M.R.P. No. 834.

For No. 834, *read* No. 854.

Proceedings at meetings of the Society

Special General Meeting, 16 June 1954

A Special General Meeting of Fellows was held to elect the Officers and Council for the year 1954-5.

Fellows had been circulated on 10 April with the list of nominations for Officers and Council for the year 1954-5. No other nominations having since been received, the persons so nominated were declared to be elected unopposed in accordance with By-Law 51.

The Officers and Council for 1954-5 were therefore elected as follows :

PRESIDENT

OLIVER GRAHAM SUTTON, C.B.E., D.Sc., F.R.S.

VICE-PRESIDENTS

Professor PERCY ROBERT CROWE, PH.D.
SIR CHARLES NORMAND, C.I.E., M.A., D.Sc.
CAMERON DARRELL OVEY, B.Sc., F.G.S.
Professor PERCIVAL ALBERT SHEPPARD, B.Sc., F.INST.P.

VICE-PRESIDENT FOR CANADA

FRANK WALDEMAR BENUM, M.A.

VICE-PRESIDENT FOR SCOTLAND

ALEXANDER EBENEZER MCLEAN GEDDES, O.B.E., M.A., D.Sc.

TREASURER

JAMES STRACHAN FARQUHARSON, M.A., D.Sc.

SECRETARIES

HOWARD LATIMER PENMAN, M.Sc., PH.D., F.INST.P.
GEORGE DAVID ROBINSON, PH.D., F.INST.P. (Librarian).
RICHARD GEORGE VERYARD, B.Sc.
THOMAS WILSON WORMELL, M.A., PH.D. (Editor).

COUNCILLORS

CHARLES JAMES BOYDEN, B.A.
GEORGE ARTHUR BULL, B.Sc.
HENRY CHARNOCK, M.Sc., D.I.C., A.INST.P.
JAMES MARSTON CRADDOCK, M.A.
RONALD FRITH, O.B.E., PH.D.
RALPH EDWIN LACY, B.Sc.
FRANK HENRY LUDLAM, D.I.C.
Instr. Lt. THOMAS ARTHUR MARSHALL, B.Sc., R.N.
FRANK PASQUILL, D.Sc.
Professor NORMAN PYE, B.A.
RICHARD SEGAR SCORER, M.A., PH.D.
MRS. ANN COURTENAY WELCH, M.B.E.

Ordinary Meeting, 16 June 1954

The following candidates were declared to have been elected that day by the Council :

FELLOWS

ELLWOOD, Thomas William, 59 Stonecross Road, Hatfield, Herts.
GHOSH, Capt. Bijoy Kumar, 123b Manohar Pukur Road, Calcutta 23, India.
MOORE, John Martin Albert, 23 Exeter House, Bishops Bridge Road, London, W.2.
MORRIS, George William, F.R.G.S., F.Z.S., 8 Buck Lane, Sale, Manchester.
OLIVIER, Henry, C.M.G., M.Sc. (Eng.), PH.D., M.I.C.E., P.O. Box 345, Jinja, Uganda.
TICKNER, Peter Edward, The Bugle Flat, Hamble, Hants.

FOREIGN MEMBER

WOODWARD, Miss Elizabeth Woods, Riderwood, Maryland, U.S.A.

ASSOCIATE

WILSON, Robert Henry, c/o West African Airways Corporation, Airway House, Ikeja, Lagos, British West Africa.

STUDENT MEMBER

WHITEROD, James, 3 West Cottages, South Benwell, Newcastle-on-Tyne 5.

Dr. R. C. Rainey of the Anti-Locust Research Centre, gave a short talk on ' Observations on flying locusts as evidence of air movements on different scales.'

The following papers were read and discussed :

(*a*) ' Computation of tendencies and vertical motion with a two-parameter model of the atmosphere,' by F. H. Bushby and Mavis K. Hinds (*Q.J.*, **80**, p. 16).

(*b*) ' The computation of forecast charts by application of the Sawyer-Bushby two-parameter model,' by F. H. Bushby and Mavis K. Hinds (*Q.J.*, **80**, p. 165).

(*c*) ' The evaluation of infra-red radiative fluxes due to atmospheric water vapour,' by W. L. Godson (*Q.J.*, **79**, p. 367). The paper was presented by Dr. G. D. Robinson.

Summer Meeting, 7 July 1954

The Summer Meeting of the Society was held on 7 July 1954 at the National Institute of Oceanography, Wormley, near Godalming, by kind permission of the Director, Dr. G. E. R. Deacon, F.R.S. Some 80 Members and guests attended and enjoyed a very interesting and instructive afternoon spent in inspecting the many ingenious exhibits in the building. At the close of the meeting a vote of thanks was moved by the Society's President, Dr. O. G. Sutton, C.B.E., F.R.S.

Two-day Summer Meeting, Edinburgh, 15-17 July 1954

The first two-day Summer Meeting to be arranged by the Society was held in the Department of Natural Philosophy, Edinburgh University, by kind permission of the Principal, Sir Edward Appleton, F.R.S. The meeting consisted of two scientific symposia.

The first on ' The upper atmosphere,' with the Society's President, Dr. O. G. Sutton, C.B.E., F.R.S., in the Chair, was opened by Sir Edward Appleton, F.R.S. Contributors were : I. C. Browne (Jodrell Bank), ' Winds, temperatures and densities in the region 80-100 km '; H. P. Palmer (Jodrell Bank), ' Winds in the F-region from scintillation of radar stars '; D. H. Johnson (Dunstable), ' Solar tides in the lower stratosphere '; D. H. McIntosh (Eskdalemuir), ' Upper winds from geomagnetic observations '; A. H. Jarrett (St. Andrews), ' Temperature measurements from auroral spectra '; J. Paton (Edinburgh), ' Aurora and noctilucent clouds.'

The second symposium was on ' Meteorology and Scottish agriculture,' with Dr. A. E. M. Geddes, O.B.E., M.A., Vice-President for Scotland, in the Chair. Contributors were : J. Grainger (Auchincruive), ' Relation of climate to plant disease '; J. Grainger, J. L. Sneddon, and E. C. Chisholm (Auchincruive), ' Climate and the yield of cereal crops '; F. L. Waterhouse (Dundee), ' Microclimatology of grass '; J. R. H. Coutts (Aberdeen), ' Soil temperature in an afforested area in Aberdeenshire '; J. M. Caborn (Forestry Department, Edinburgh), ' The influence of shelter belts on microclimate '; F. H. W. Green (Nature Conservancy), ' The climatological work of the Conservancy.'

In addition a very enjoyable excursion to Eskdalemuir Observatory was arranged, attended by the President and some 70 Members and Guests. Inspection of the Observatory was followed by tea when the thanks of the Society were expressed by Prof. P. A. Sheppard, Vice-President.

At the close of the meeting Members were taken on a conducted tour of the Royal Mile, led by Mr. J. Hossack, Director of Extra-mural studies, Edinburgh University, which was followed by tea at the Sanctuary, Holyrood House.

Council wish to record their sincere gratitude to the Secretary of the Scottish Centre, Mr. J. Paton, who was responsible for making the local arrangements.

Reviews

Mathematics in action. By O. G. Sutton. London (G. Bell), 1954. Pp. viii, 226; 52 Figs. 16s.

During the past fifty years many books have been written by pure mathematicians which have revealed the accumulated wealth of ideas and methods in mathematics but in this book we encounter an applied mathematician bent on a similar task. The subject matter and point of view are accordingly altered. The subject matter is so wide that, in a book of this size, a personal choice must be made of the topics to be included and Dr. Sutton chooses those topics which he himself has encountered in many and varied spheres. The task he sets himself is indeed formidable ; the explanation or revelation of the idea in conformal representation, or in a Fourier integral, requires not only considerable depth of knowledge and perspective but more than all, in a book of this kind which will have a popular appeal, it requires a happy choice of words. It is at once evident in the first chapter that the necessary and sufficient qualities are at the author's command; and the whole book is a model of lucidity and here I would particularly like to mention the beautifully written chapter on Waves.

The main topics dealt with are as follows, Ballistics, Waves, Aerodynamics, Statistics and the Weather, but in order to prepare the reader for the ascent to these heights there are two chapters which are devoted to elucidating the type of problem which the applied mathematician has to pose and the tools which are available for the task. The question of emphasis obviously is of the first importance and here one's personal inclinations play a dominant role. In Chap. 2 where differential and integral calculus are elegantly discussed I felt that the subject of vectors might have been accorded more space; there seems little reason to treat the operator *curl* as a Cinderella when *div* and *grad* have been given preferential treatment, and likewise on p. 70, Eq. (1), the statement of the equation would be more acceptable in the bold type of the vector notation. In the following chapter on Ballistics the classical theory is reviewed and the standard numerical technique is discussed for the calculation of trajectories. This chapter ends with a clear account of the differential analyser; following this I would have welcomed a much longer section on digital computers. The chapter on Waves, as mentioned above, is quite outstanding; so also is the next chapter called not Aerodynamics but, more appropriately, the Mathematics of Flight. The chapter on Statistics is at once concise and clear and would form an admirable first reading course for the technical student who desired a rapid acquaintance with the field. In Chap. 7, Mathematics and the Weather, attention is directed simply to the equations of motion, to an understanding of the geostrophic approximation and finally to the idea of numerical forecasting.

Most applied mathematicians will enjoy this book; it will be of value to the non-mathematician not only in explaining the point of view of the applied mathematician but also in introducing him to concepts which might otherwise be for ever veiled in obscurity. I would like to add one final word about the diagrams which are well and clearly executed.

T.V.D.

Physical meteorology. By John C. Johnson. New York (John Wiley), 1954. Pp. xii, 393; 133 Figs., 36 tables. 60s.

This text-book of ' meteorological phenomena not directly connected with the circulation of the atmosphere,' is of a standard suitable for those taking a degree course in physics or mathematics, but having no specialized meteorological knowledge. The subjects discussed fall into five rather disconnected groups, and the book seems to lack a central theme. This lack may follow automatically as soon as reference to the circulation of the atmosphere is excluded from a meteorological text and the book should prove useful to physicists and geophysicists who require introductions to any of the subjects mentioned below. The chapters are generally clear and accurate, and are followed by useful references to recent papers, and to more specialized ' source books.'

The first three chapters contain a good though long discussion of atmospheric refraction, scattering and visibility, and chap. 6 deals with the physics of haloes and coronas, and the subjects usually termed atmospheric optics. Chapters 4 and 5 on the long- and short-wave radiation processes in the atmosphere, and the heat budget of the earth, would form a useful and adequate introduction to this active branch of meteorological research.

Chapters 7 and 8 are concerned with the formation and growth of cloud particles, and the precipitation processes in clouds. The discussion is full and accurate, and the limits of our present knowledge are stated clearly. These chapters also include a valuable discussion of the quantitative use of radar in cloud studies, and a brief but suitably cautious discussion of cloud seeding and the artificial release of precipitation.

Chapter 9 summarizes work on atmospheric electricity and thunderstorm electrification, and the last two chapters compress into 60 pages discussions of the ozonosphere, the ionosphere, and the measurements of temperature, density, pressure and humidity in the upper atmosphere. These two chapters seem rather brief for the subjects covered, and perhaps an even more useful book would result from an extension of these chapters at the expense of the 130 pages devoted to visible radiation in the atmosphere. This book will be, none the less, a very useful addition to any meteorological bookshelf, for it makes readily available in text-book form information, on a number of topics, which has hitherto been secluded in specialist books and papers.

The book, pleasantly bound and printed, is very liberally supplied with clear figures and tables, and the reviewer is especially pleased to see the series of problems given at the end of each chapter.

H.P.P.

Witterung und Klima in Mitteleuropa. By Dr. Hermann Flohn. Stuttgart (S. Hirzel), 2nd Edition 1954. Pp. 214; 33 Figs., tables, plates. 24 DM.

The first edition of this distinctive work under a slightly different title (*Deutschland* instead of *Mitteleuropa*) first appeared in 1942 and may not have received wide notice outside Germany. Reset in roman type with interesting additional chapters on Austria by Dr. F. Lauscher and on Switzerland by Dr. M. Schüepp and an extended bibliography, the book retains its unique character as a discussion of the mechanism of a regional climate in terms familiar to the synoptic meteorologist. This aspect of climatology, which has in relatively recent years become known as synoptic or dynamical climatology, is termed by Professor Flohn ' modern climatology ' while the more established studies of the climatic elements separately, which give special prominence to normal values, are termed ' classical climatology.' The terminology will not be readily accepted by everyone for the words ' modern ' and ' classical ' in no way suggest the different methods of treatment both of which are necessary and useful and will continue to be so. Indeed for most practical purposes of applied climatology the ' classical ' method remains the more valuable although it is scientifically somewhat arid. No doubt the climatological study of synoptic behaviour, weather types, air-mass frequencies, frontal passages and the like is important for an understanding of climate and is particularly useful as background knowledge for the synoptic weather forecaster but it does not provide ready answers to most of the questions put to the climatologist by his public.

The reader who is interested in the usual climatological statistics will not therefore find the information in this book which has very few tables and no climatic map. But these things can readily be found elsewhere and there is no doubt that this discussion of the structure of climate is a valuable supplement. An interest in the climate of central Europe is indeed hardly necessary to make Professor Flohn's book worth acquiring, for he is as much concerned with ideas and methods of general meteorological interest as with facts peculiar to central Europe.

The subject matter does not lend itself readily to formal text-book presentation and the different chapters are rather in the nature of separate essays. The style tends to be discursive with numerous digressions and the 184 pages of text, almost unrelieved by tables or diagrams, are likely to present difficulty to all but the fluent reader of German. The continual reference to original papers is also a distraction and one may wonder whether a bibliography running to some twenty odd pages with over 500 entries would not have been more useful had it been more selective. There are no fewer than 191 references for the study of ' singularities,' presenting a formidable prospect to the student who may wish to pursue this subject.

The first chapter is of an introductory and historical nature stating the author's attitude to climatology. The remainder deal amongst other things with weather types, air masses, large-scale weather types (*Grosswetterlagen*) and climatic oscillations, but perhaps the most striking chapter is that in which the author takes us on a conducted tour through the year pointing out the characteristics of the seasons with special attention to singularities and regularities. It is a difficult art to make a coherent story from the synoptic weather of the year in the zone of the ' turbulent baroclinic westerlies ' and it requires almost an act of faith to recognize as many as seven ' waves ' to the early summer ' monsoon.' It is unlikely that the sceptic will be persuaded to accept this analysis from the evidence provided and until the phenomena have been more strictly defined and statistically tested there will remain those who regard most ' singularities ' as chance coincidences in the past with no prediction value for the future. But the subject has received relatively slight attention outside Germany and is much too important to be neglected.

Mention must also be made of the excellent diagrams collected at the end of the volume, especially those giving the long-term mean values or frequencies for each day of the year of days with föhn, air-mass change, cloudy and sunny days, sunshine duration, precipitation, thunderstorms, pressure and temperature at selected stations. One may question the validity of identifying the many peaks and troughs of a few days' duration with entities of climatological significance, but the facts are striking and it should be the aim of ' modern climatology ' to give them a satisfying objective explanation.

R.C.S.

On the origin of the solar system. By H. Alfvén. International series of monographs on physics, Oxford (Clarendon Press), 1954. Pp. viii, 194; 47 Figs., 15 tables. 30s.

In this book a novel theory of the origin of the solar system is discussed. It is a theory that does not require the intervention of other celestial objects for generating the sun, planets, and satellites, but merely the interaction of the sun with infalling gas and dust clouds. The author, who has played a considerable part in impressing the importance of magnetic interactions on astronomical thought, makes these play a large part here also; and he is no doubt on strong ground there.

The tracing of the main processes of the theory is, however, not convincing. According to his theory, the materials falling towards the sun have to be suspended against gravity for very long periods of time in order that diffusive separation of chemicals may take place. A magnetic field is postulated as causing such suspension, but a field can only do this if it is firstly strong enough and secondly if its lines of force do not link the gravitating body. The required strength is 10^6 times greater than the estimates of the present solar magnetic field, and this assumption is discussed. But the completely vital question of linkage is not discussed, and the field is treated as a solar dipole. A reference is made to this basic conflict but no resolution is suggested.

The well-known cosmogonical theory of the distribution of the angular momentum within the solar system is also treated only sketchily. No detailed mechanism of the transfer of this from the sun to a surrounding gas cloud is given, and one may wonder whether the assumption of a strong solar magnetic field is really all that is required.

Despite the severe criticism that can be raised against the theory as a whole, there is no doubt that the book contains stimulating discussions of various points of magnetodynamics which may well be of great importance in a future theory.

The index is somewhat inadequate; for example, of the seven authors mentioned on p. 1, only four appear in the author index. The quality of the printing, the plates, and figures is of the usual high standard of the Oxford University Press.

Thomas Gold

Dynamik der Zyklonen. By P. Raethjen. Leipzig (Akademische Verlag., Geest and Portig), 1953. Pp. xii, 384; 92 Figs., 3 tables. 35 DM.

This is not so much a text-book as an up-to-date report on researches made in all schools of meteorology. It covers a wider field than a narrow interpretation of its title would imply, perhaps inevitably, since the dynamics of cyclones could hardly be dealt with adequately without bringing in considerations of the general circulation. In a critical historical review the author gives an exposition of many early ideas which appear in proper perspective against the background of all the work done and he extracts from earlier work ideas which still appear valid in the light of more recent and comprehensive observations. In this way he bridges the gap between earlier and more recent theories but any forecaster who looks to find a single model of a depression emerging as a building erected on solid foundations will be disappointed. No such model exists but numerous possible causes of depressions are clarified.

The book is divided into two parts, each with seven chapters, the first being descriptive, and the second mathematical. The first part is not only descriptive, but attempts to give the physical ideas underlying the mathematical treatment later in the book. The first two chapters deal with the old basic conceptions in a somewhat critical manner, regarding polar front waves not as boundary waves but as disturbances of a widespread state of unstable equilibrium. Only partially then is the wave theory of cyclone development retained. In the third chapter the development of frontal zones is discussed in the light of the accumulating mass of aerological observations and the interaction between depressions and the mean field of motion is clearly described as well as the 'jet-stream,' 'cut-offs' and 'meandering.' The general circulation is discussed in chap. 4, a meridional cross-section demonstrating a type of circulation which will not be acceptable to all.

In chap. 5 the chief possibilities of development, from vorticity considerations, are discussed, first among those being the maintenance of absolute vorticity in meridionally moving air masses. In the remaining two chapters of the first part of the book different models of depressions are discussed and some forecasters may be surprised by the suggestion that high-level depressions may be formed by sinking of the polar stratosphere. Other suggestions in these chapters appear to be debatable.

In the mathematical half of the book the author plunges straight into vorticity and consideration of its variations, while in the following chapter the importance, for the latter, of horizontal divergence and vertical velocity are dealt with. Chaps. 10 and 11 include discussions of the pressure changes due to the vorticity changes and these chapters on vorticity have a disarming simplicity about them. A discussion of large-scale turbulence is given in chap. 12, in relation to frontal zones, so as to explain the development of centres of action, and in chap. 13 the long waves in the zonal circulation (after Rossby) are discussed.

The closing chapter deals with the convectional depression with particular reference to tropical hurricanes.

Some of the ideas of Professor Raethjen will appear strange and perhaps bizarre to British synoptic meteorologists but for those in search of a new approach to the problems of synoptic meteorology this may be intriguing if not stimulating. In these days when synoptic ideas have to be shaped to suit machines there may be a danger of machine-made ideas. There is a novelty about some of the things which this author writes which might, even indirectly, avert this danger.

The book is beautifully printed, though the binding may not suit British taste, and there is a bibliography comprising 840 items.

J.S.F.

Seismology. By K. E. Bullen. London (Methuen), 1954. Pp. viii, 132; 15 diagrams. 8s. 6d.

Seismology is nowadays a vast subject, and the literature is still extending rapidly. To give an adequate account of the technique, the theory and the inferences that can be drawn from earthquake studies requires the nicest discrimination, as well as an author completely *au fait* with the latest developments. Prof. Bullen, whose researches have contributed so largely to our present knowledge of the earth's interior, is exceptionally well qualified for this task, and this new book is a marvel of compression, yet nowhere has clarity been sacrified in the interests of brevity.

The treatment is quantitative so far as that is reasonable and practicable, and some saving of space is effected by omitting detailed mathematics and giving appropriate references to Prof. Bullen's standard treatise on theoretical seismology. The non-mathematical reader, however, need not be put off; he will

in any case desire information as precise as can be given concerning, for instance, the dimensions and physical properties of the core of the earth, and it suffices to take the skeleton mathematical outline for granted, much as any mathematician in fact would do. What is important is the chain of inferences connecting the data of observations.

In a brief review it is at most possible to indicate only a selection of the topics covered. A succinct summary is given of the early history of seismology and of the experimental technique of recording shocks and locating their epicentres. The interpretation of records in terms of elastic waves (compressional and distortional) through the body of the earth and the propagation of elastic waves over the surface calls for an outline of the theory of elastic waves, but it is the determination of ray paths through the earth that is of real interest to the geophysicist, and it is with this topic that much of the author's research work has been concerned, especially in finding the variation of density and elastic properties at all depths within the earth.

The region of the earth concerning which most uncertainty exists is the ' inner core,' extending from a depth of 5,120 km to the centre, and the direct findings of seismology need supplementing by an arbitrarily assumed distribution of density; in any case, a jump in density of about 4 g/cm^3 seems to be required at the boundary of the inner core, and the density at the centre is taken rather arbitrarily as 17·5 g/cm^3. This constitutes Bullen's ' Model A.' Bullen noticed, however, that although the density and rigidity are widely different on the two sides of the boundary (at a depth of 2,900 km) between the mantle and the ' outer core,' the compressibility is nearly continuous, and, if the layer 200 km thick just outside the core is ignored, the same is true of the rate of variation of compressibility with pressure. This suggests that at high pressures the compressibility is a function of the pressure only. Using this ' compressibility ' hypothesis, Bullen has constructed earth ' Model B,' in which the density, increasing with depth through the main body of the earth's mantle, jumps from about 5·7 to 9·2 g/cm^3 at the outer boundary of the core, while at the junction of the outer and inner cores the density increases from 12 to 15 g/cm^3 in a distance of 140 km, with a more gradual increase to nearly 18 g/cm^3 at the centre. A discussion of Model B suggests that the inner core is solid, and not liquid like the outer core. The treatment involves an outline of Birch's recent work.

This little book, though compact, is thoroughly up-to-date; the author, for instance, has found space to include a sketch of the work of Longuet-Higgins on microseisms and the Macelwane-Ramirez method of tripartite stations. In conclusion, Prof. Bullen has written a paragraph on each of twenty earthquakes, selected mainly for their seismological interest or for their stimulus to the development of the science. The book is well printed and remarkably free from misprints; it will be valuable alike to specialist and to the general reader.

R. Stoneley

Reports on progress in physics, Vol. XVII. Editor A. C. Stickland. The Physical Society, London, 1954. Pp. 280; Figs., tables. 50s.

The writer looks forward to the annual publication of this volume with pleasant anticipation, and once again expectations are fulfilled with a lively and authoritative series of essays on eight fields of modern physics. It has now become essential reading for any physicist who wishes to keep acquainted with modern developments, and for those who are engaged in atmospheric studies it once more contains some matter of specialist interest.

Any attempt to comment upon ' Nuclear shell structure ' by M. H. L. Pryce, ' Atomic valence states and chemical binding ' by W. Moffitt, or ' Antiferromagnetism ' by A. B. Lidiard would only expose the writer's ignorance; suffice it to say that they make interesting reading. ' Diffraction theory ' by C. J. Bouwkamp is interesting in the evidence it provides of a revived interest in certain branches of classical physics. For those who are interested in droplet scattering there is a very useful list of references to recent work on Mie's theory, although the theory itself is not discussed in detail. Incidentally a coming book on scattering by Van de Hulst is mentioned which will be most welcome.

Two interesting astronomical articles are ' Theory of the origin of cosmic rays ' by Edward Teller and ' The physical condition of the solar corona ' by C. W. Allen. Astronomy and Geophysics have a long association, and these two articles should be of real interest to members of the Society, particularly in view of the great importance which is now attached to understanding the dynamics of stellar systems.

Finally, there are two articles entirely concerned with the earth's atmosphere. ' Atmospheric electricity ' by J. Alan Chalmers is a comprehensive review which emphasizes results obtained since the author's book on the subject was published in 1947 or which were not available at that time. As the writer remarks, there is considerable overlap with Wormell's review in the *Quarterly Journal*, but a difference of view-point is always valuable.

' Horizontal movements in the ionosphere ' by B. H. Briggs and M. Spencer is necessary reading for all meteorologists, since here are discussed many recent measurements on the E- and F-layers which give broadly consistent results, giving at last a reasonably firm basis for studies of the dynamics of the ionosphere. The writers stress that the use of the word ' movements ' rather than ' winds ' is deliberate, since it is still arguable that the motion of ionization is not the same as the motion of the air at high levels. However one is clearly entitled to think in terms of air motions until much more potent objections can be put forward. The article is mainly concerned with the method of ' closely-spaced receivers ' which makes use of the correlation existing between the fading patterns at two close receiving stations, and fairly consistent results for diurnal and seasonal variations from stations in England, Canada, U.S.A. and New Zealand are

given. These results are confirmed by measurements of the movements of large-scale irregularities in the F-region, measurements of the scintillation of radio stars and measurements of the motions of ionized meteor-trains. Finally mention is made of six non-radio methods of detecting ionospheric motion, although these do not include the possibility of observing smoke puffs from high-altitude rockets. This is all first-class observational work. It has not been possibly to discuss the results here, but the reader is strongly recommended to read about them himself.

R.M.G.

The restless atmosphere. By F. K. Hare. Hutchinson's University Library, 1953. Pp. vii, 192; 17 Figs., tables. 8s. 6d.

In recent years there has been a marked development in the scientific approach to the problems of meteorology but the advance in the application of physical principles has been less rapid in some branches of meteorology than others. Judging from the textbooks, with some notable exceptions, progress in physical understanding in the field of climatology has lagged – possibly because so many of the workers in that field have had no direct interest in meteorology.

The author, now Professor of Geography at McGill University, has been a practising meteorologist and in the light of the experience and knowledge which he gained whilst serving in the Meteorological Office he has endeavoured in a modest way to explain types of world climate by describing the weather processes involved – without the use of mathematics !

The first eight chapters give an able and fairly detailed account, with the aid of some elementary physics, of the energy budget of the atmosphere, the formation of clouds and precipitation, air motion, fronts and air masses, pressure patterns and, finally, the general circulation of the atmosphere. In regard to the last item one would like to have seen some diagrams showing the distribution of upper winds and some reference to the well known phenomenon of ' blocking action ' but it seems that space was limited.

The next six chapters deal with regional climates on the basis of the physical understanding which the reader is expected to have acquired from the earlier part of the book. In this second very interesting part the author has tried to pour a quart into a pint pot and has had to make some broad generalizations. However, we are informed in the preface that the regions treated in detail are those generally covered by university courses in regional geography for honours students in Britain and North America. Certainly much of the southern hemisphere is rather neglected especially, as the author admits, the temperate belts and to be told that he has nothing to add to what has already been written is rather disappointing. It is only fair to add, however, that the author concludes with a chapter giving advice on further reading plus a list of references which should enable the student to make a more detailed study of what is so appropriately called the restless atmosphere.

The author states in the preface that much of the material in the chapters on regional climates is controversial and many will share that view. For example, meteorologists with experience in the region regard the haboobs of the Sudan as the visual evidence of the micro-fronts of thunderstorms rather than of ' waves in the easterlies.' Nevertheless, the intention was to stimulate interest in ' the no-man's-land ' of dynamic climatology ' and in this it can be said that the author has achieved a considerable measure of success.

It would have been helpful if a list of illustrations had been given as some of the figures are widely separated from the reference in the text, but this is a minor criticism of what is otherwise quite an attractive little book. Compared with the cost of similar publications it is good value for 8s. 6d. – not only to geographers but to meteorologists also.

R.G.V.

Klima und Wetter in Arosa. By F. W. Paul Götz. Frauenfeld (Huber & Co.), 1954. Pp. 148; 65 Figs., 64 tables. Fr. 18.70, DM. 18.

Arosa is a well-known ski-ing and holiday resort in a high valley in eastern Switzerland. The observatory is situated a little above the bottom of the valley, at a height of 1,854 m (6,083 ft) above sea level. The observations at the present site go back to 1900 and on a neighbouring site to 1890. Recently radiation and ozone observations have been made with modern apparatus.

The local climate is fully discussed. Owing to its height and its position in the interior of the mountain range, Arosa is largely free from winter stratus and stratocumulus. Mean relative humidity is as low as 60 per cent in February and is only 65 per cent for the whole year; in July-September it is 69 per cent. Cloudiness is greatest on summer afternoons and the mean for 0730, 1330 and 2130 hr is higher in summer than in winter. Precipitation has a pronounced summer maximum, and even fog is most frequent from May to October. Snow can lie in any month, but in July the frequency is only one day in two years. The lasting snow-cover starts some time between early October and about 20 December, and ends between early March and late May. The average depth is just over a metre in March.

There are some attractive photographs of mountain clouds, and two showing sun pillars below the observer, a not infrequent phenomenon in the Alps.

Though the book deals mainly with local climate, there is some material bearing on larger problems such as singularities and long-period climatic changes.

C.K.M.D.

Automatic digital calculators. By A. D. Booth and K. H. V. Booth. London (Butterworth), 1953. Pp. vii, 230; Figs., plates. 32s.

The authors of this book are respectively Director and a member of the staff of the Electronic Computation Laboratory at Birkbeck College, London. They both had experience in the United States immediately after the war during the years in which the logical principles of the design of computers were being rapidly developed. The book contains a general account of the construction and use of digital computers. Naturally in the space of some 200 pages all aspects of these topics cannot be treated in detail but the book will serve admirably as a general introduction for a newcomer to the subject and will also enable those who have had some contact with a particular computing machine to broaden their knowledge. Readers whose interest is mainly in the use of computers and who are only interested in logical design in its broadest aspects may find the book rather biased towards the detailed design of circuits although they will find in the later chapters discussion of such topics as the use of Chebyshev polynomials for the representation of functions. The following list of chapter headings gives an idea of the contents of the book : introduction; the mechanical era; the advent of electronic techniques; the overall design of a computing system; the control; the arithmetic unit; miscellaneous operations; input-output; gates; single-digit storage; miscellaneous components; storage devices; definition of a code and discussion of its form and contents; the technique of coding; the use of sub-routines in coding; programme design; some applications of computing machines. There is also a full bibliography.

M. V. Wilkes

Obituary Notices

Lt.-Col. T. L. ORMISTON, M.A.

Thomas Lane Ormiston, of Trood House, Alphington, Exeter, who had held Fellowship in the Society since 1930, died on 19 June 1954. Born in 1867, he received his education at Trinity College, Oxford, and the Royal Military College. Later he was admitted barrister at the Inner Temple. He served with distinction in the Indian Army and in various civilian capacities at home and abroad, being for many years a member of the Devon county council and a justice of the peace. In 1935 he was appointed a general commissioner of income tax for Great Britain, and in 1940 became high sheriff of Devon.

E.L.H.

Mr. W. C. REYNOLDS, B.Sc.

A promising career was cut short by the death in a road accident on 1 August 1953 of Walter Charles Reynolds, at the age of 32. Mr. Reynolds joined the staff of the Meteorological Office as a junior assistant in March 1939 and after periods of training and service at Bircham Newton and White Waltham was commissioned to the R.A.F.V.R. in April 1943. When the war ended he was sent to India for three years. Returning to the Meteorological Office in England on his release from the R.A.F.V.R., he was promoted to the rank of experimental officer in July 1948. Subsequently, in 1950, he became forecasting instructor at the training school. In 1952 he graduated B.Sc. after attending a four-year course of evening lectures at Birkbeck College, and was then transferred to the Special Investigation Branch of the Office. At the time of his death he was studying for the Ph.D. degree. Mr. Reynolds was elected to Fellowship of the Society in 1942.

E.L.H.

Lieut.-Comdr. C. N. WYNN, M.A., R.N.V.R.

Charles Norman Wynn, a Fellow of the Society since 1944, died very suddenly on 14 January 1954 while on holiday in Switzerland. After graduating with honours in history at Keble College, Oxford, he entered the teaching profession, wherein his appointments took him initially to Canada and Egypt. During the 1939-45 war he joined the R.N.V.R. as meteorological officer, serving in the Orkneys and at Colombo and Singapore. To him fell the high responsibility of advising the East Indies commander-in-chief on weather conditions for the landings at Rangoon and the subsequent return. When peace had been restored Lieut.-Comdr. Wynn resumed his scholastic career. At the time of his death he was head master of a preparatory school at Sevenoaks.

E.L.H.

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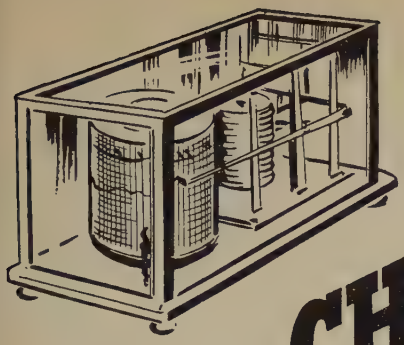

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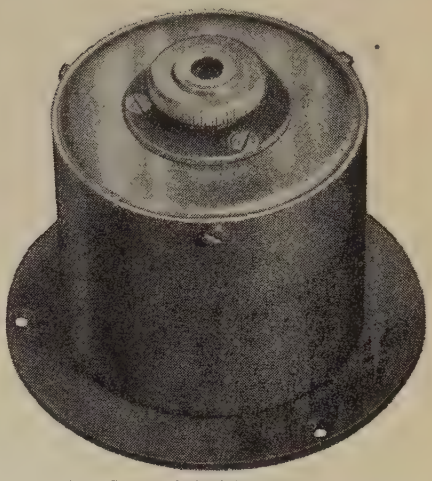

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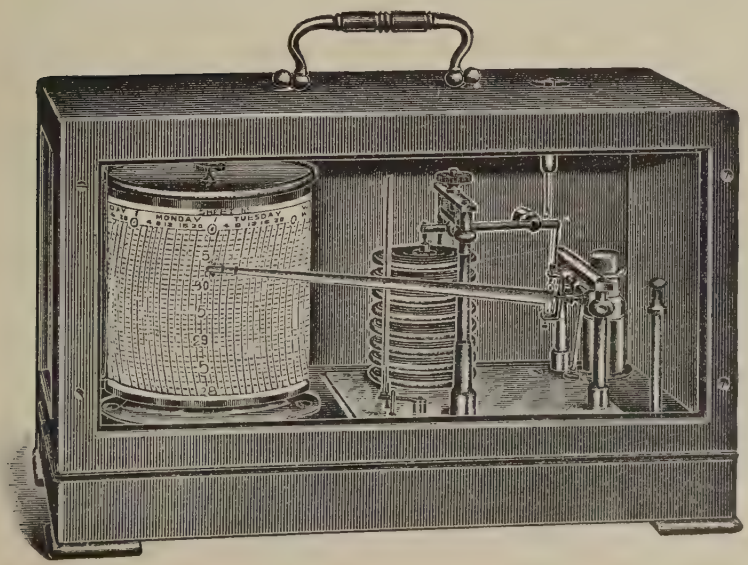

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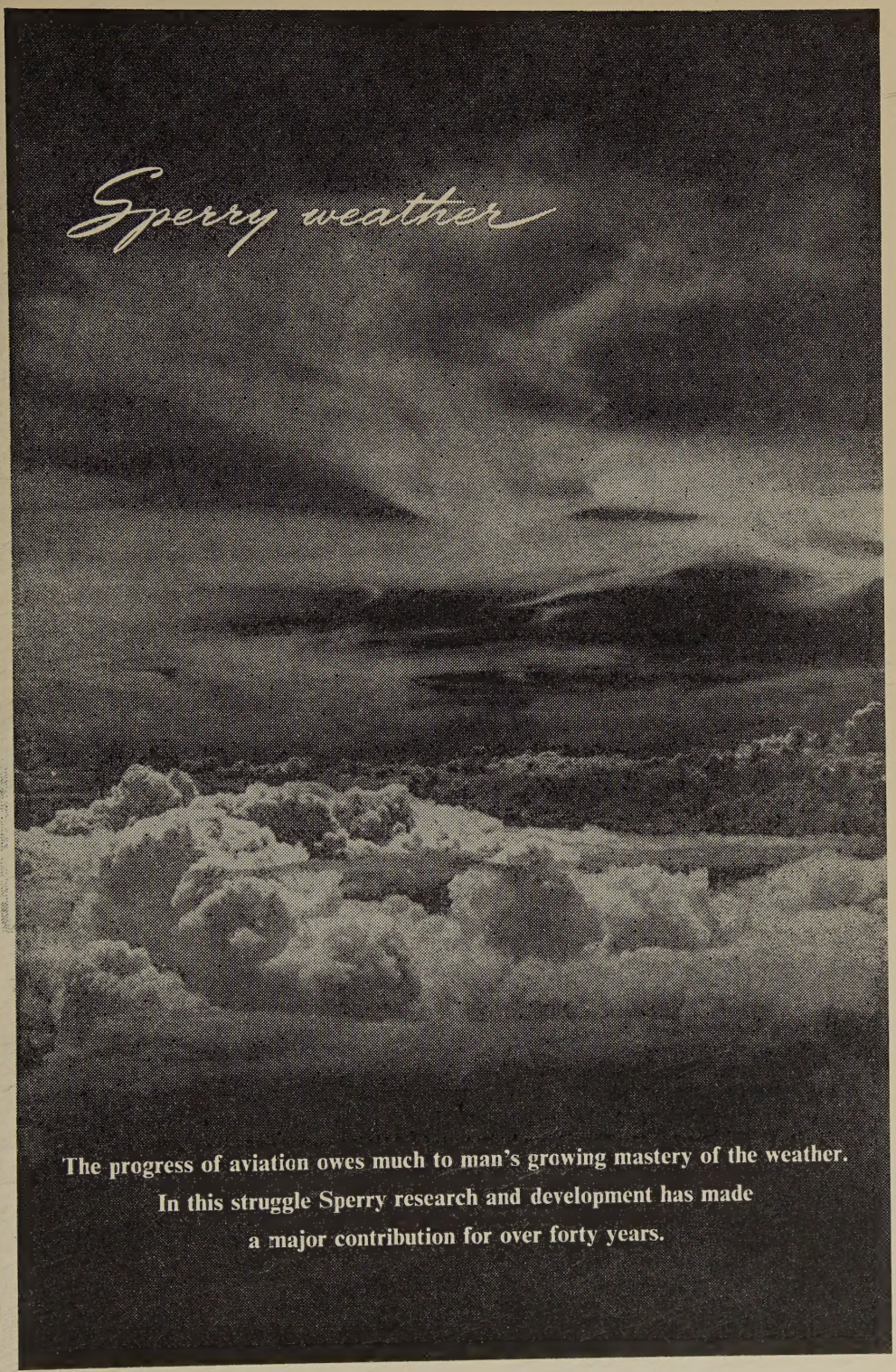
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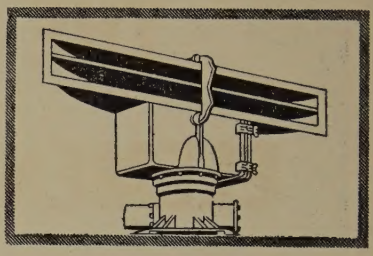

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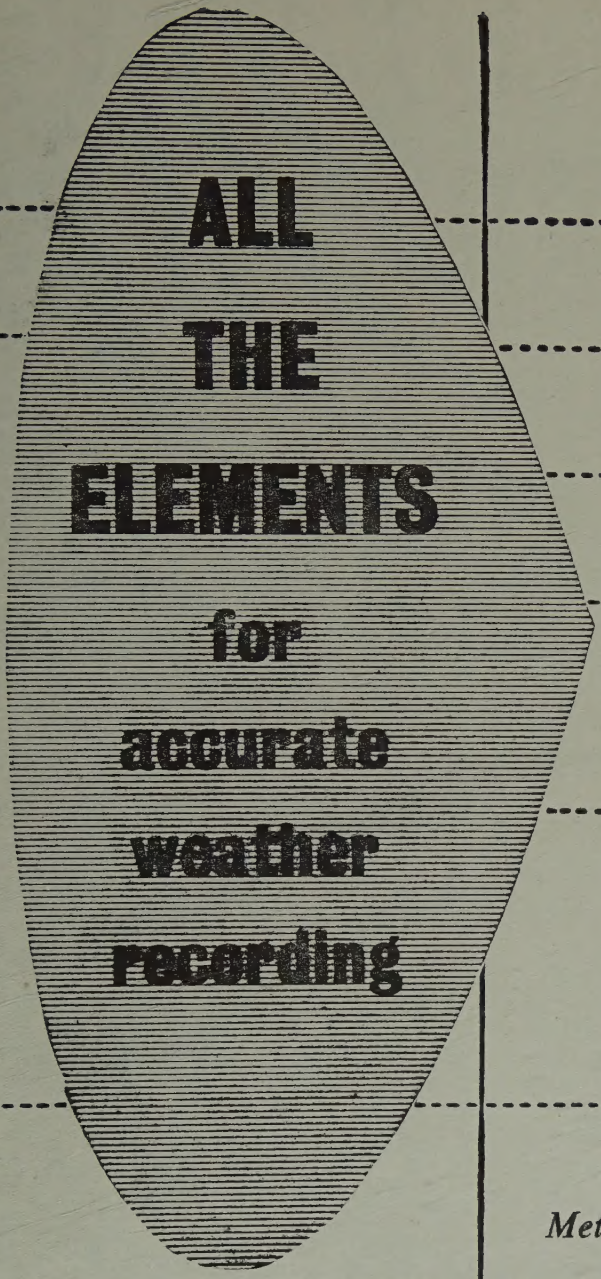

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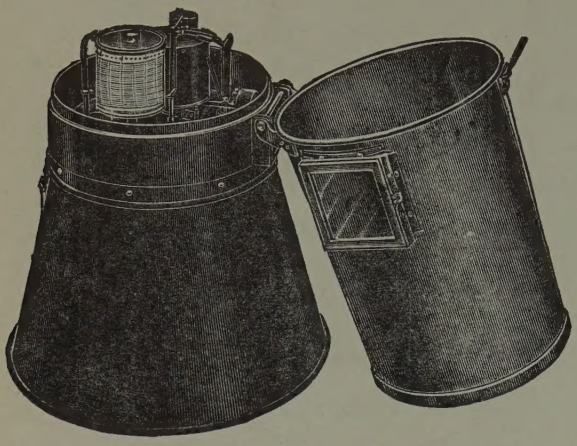

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